Contents

1 Introduction and overview

1.1 Aim

Quark and lepton mixing are important topics in modern particle physics that are not really covered in detail in the standard particle physics lectures. The central theme of these lectures will be to discuss how the elements of the quark CKM (Cabibbo, Kobayashi, Maskawa) and neutrino P-MNS (Pontecorvo, Maki, Nakagawa, Sakata) mixing matrices are measured, to compare their values and consider what the values mean for particle physics phenomena.

The lectures will therefore treat in some detail:

- Discovery and properties of the leptons, especially the neutrinos
- Neutrino oscillations and determination of P-MNS matrix parameters
- Quarks, their decays and the CKM matrix parameters
- Mixing, oscillations and CP violation in K and B systems

For the measurements with quarks, I will concentrate on physics at modern particle physics colliders, especially LEP, Tevatron and the B factories. Older data will be included as appropriate. For the neutrino oscillation measurements, use has mostly been made of atmospheric and solar neutrinos. However, more and more experiments under controlled conditions using reactors or high energy particle accelerators are coming to the fore and their results will also be discussed.

1.2 Outline

Homepage for the course is:
`http://pi.physik.uni-bonn.de/~brock/teaching/atpp_ss10`.

The course is also registered under Ilias/Ecampus:
`https://ecampus.uni-bonn.de/goto_ecampus_crs_26199.html`

Overall plan of lectures:

1. Introduction and overview
2. Leptons, neutrinos and their masses
3. Neutrino oscillations
4. CKM matrix parameters
5. Meson mixing and oscillations

6. CP violation in K system
7. CP violation in B system
8. Luminosity measurement at colliders
9. Direct searches for dark matter

Difficult to recommend one book for the whole course, as it covers fairly diverse topics. Literature recommendations (for now only really neutrino-relevant) can be found on the web-page. They will be updated over the course of the semester.

Ecampus will be used for announcements, emails, exercise sheets etc. Please register there.

Printed copies of lecture notes and slides will be made for each chapter when it is finished. A preliminary version (which is what I use during the lecture) can be found on ecampus and probably on the web-page before the start of each lecture.

1.3 Prerequisites

Assume that you have already heard or are following:

- Nuclear and particle physics lectures (5th semester)
- Particle physics lectures (7th semester)
- Collider physics lectures (parallel to these)

i.e. you know what fundamental particles exist and have an idea how they interact with each other. Also assume that you know what a Feynman diagram is and could draw the correct diagram(s) for simple processes, e.g. $e^+e^- \to \mu^+\mu^-$.

⇒ *Slide: Feynman graphs for* $e^+e^- \to \mu^+\mu^-$ *(mupair.mnf)*

My conventions for the graphs are shown here. Time is from left to right (not the same as Griffiths 1st edition – 2nd edition uses the usual convention – but adopted by almost everyone else). Antiparticles have arrow running backwards in time (follow the fermion number). Although direction of arrow in principle sufficient to indicate particle or antiparticle, I will often also give this explicitly. This process is referred to as s-channel or time-like. s is symbol for centre-of-mass energy.

If instead I consider $e^+e^- \to e^+e^-$, have to include scattering as well as annihilation diagram. Second diagram referred to as space-like or t-channel.

⇒ *Slide: Feynman graphs for* $e^+e^- \to e^+e^-$ *(bhabha.mnf)*

Mandelstam variables for a 2 body scattering process ($1 + 2 \to 3 + 4$):

$$s = (p_1 + p_2)^2 = (p_3 + p_4)^2$$
$$t = (p_1 - p_3)^2 = (p_2 - p_4)^2$$
$$u = (p_1 - p_4)^2 = (p_2 - p_3)^2$$

$$s + t + u = m_1^2 + m_2^2 + m_3^2 + m_4^2$$

u is rarely used, but s and t appear very often.

1.4 Cross-section and luminosity

Experimental cross-section given by:

$$\dot{N} = \sigma \cdot \mathcal{L}$$

where $\dot{N}$ is event rate given by:

$$\dot{N} = \frac{N_{\text{obs}}}{\epsilon t}$$

where ϵ is the acceptance or efficiency.

As experiment measures for a certain time, take total number of events during this time and integrated luminosity in order to calculate cross-section. Then:

$$N = \sigma \int \mathcal{L}\,\mathrm{d}t$$

$\mathcal{L}$ is the luminosity, usually measured in $\mathrm{cm}^{-2}\,\mathrm{s}^{-1}$. Typical luminosities can be seen in table in PDG:

⇒ *Slide: Accelerator properties (rpp2009-rev-hep-collider-params.pdf)*

Integrated luminosity is usually expressed in pb^{-1}, although the newest accelerators (e.g. PEP II and KEK-B) have such high luminosities that they use fb^{-1}. (LHC currently expresses integrated luminosity in $\mu\mathrm{b}^{-1}$, although that should change fast!). One used to be happy at e^+e^- and ep machines with 1 pb^{-1} per day and 100 pb^{-1} per year. The Tevatron is now achieving significantly above such values. During Run II they have already accumulated a total integrated luminosity of about 8 fb^{-1}. At the B-factories they are close to 100 times these values! They aim to go up by about anther factor of 10 with the planned super B factories. 10 fb^{-1} per year and a cross-section of 1 nb for the $\Upsilon(4S)$ implies 2×10^7 B mesons per year being produced at such machines – hence the name B factories! Often when people talk about luminosity they mean integrated luminosity.

Come back to how luminosity is measured in Chapter 7.

1.5 Quarks and leptons

Could just start with the discoveries of the leptons, especially the neutrinos, but think it is more useful to first give an overview of where we think we stand today. Quite a remarkable achievement that one can put the particle content of the standard model on a single slide:

⇒ *Slide: The Elementary Particles (chapter01_figs.odp)*

The quark and lepton masses are indicated in the table.

The masses of the quarks shown graphically are quite striking:

⇒ *Slide: Quark and lepton masses without the top quark (masses1.mnf)*

⇒ *Slide: Quark and lepton masses with the top quark (masses2.mnf)*

Charged lepton masses are well defined and measured as they only participate in electroweak interactions and exist as free particles with a reasonably long lifetime. From the observations of neutrino oscillations one infers that neutrinos are not massless. However, their masses are not known – neutrino oscillations only tell us about the mass differences between the different types of neutrino. Quark masses are much harder to define, except for the top quark, as they are always observed in bound states where the binding energy is typically larger than or similar to the quark masses.

1.6 CKM matrix

The CKM matrix arises because the quarks have mass. This also means that if neutrinos have mass one also has a CKM matrix for them, the P-MNS or MNS matrix) for leptons. It arises because the flavour eigenstates (or one can call them the mass eigenstates which have a definite flavour) are not necessarily the same as the weak eigenstates, which are the states that actually decay weakly.

I will first discuss the case of 4 quarks and leptons (i.e. 2 families), before generalising the discussion to 3 families.

1.6.1 Cabibbo angle

Over many years the weak interaction was studied and it was observed that leptons and quarks participate in so-called charged current interactions through $(V - A)$ current that couple:

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix} \quad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix} \quad \begin{pmatrix} u \\ d \end{pmatrix}$$

with a universal coupling strength, G_F. It was also confirmed that this coupling strength is valid for tau decays and is presumably also valid for the other quark families.

However, one also knows that the decay $K^+ \to \mu^+\nu_\mu$ occurs. This implies that the strange quark must in some way couple to a $\bar{u}$ quark. If we do not want to give up the concept of universality we have to come up with another idea. Assume that the charged current actually couples to the rotated states:

$$\begin{pmatrix} u \\ d' \end{pmatrix} \quad \begin{pmatrix} c \\ s' \end{pmatrix}$$

where

$$\begin{aligned} d' &= d\cos\theta_C + s\sin\theta_C \\ s' &= -d\sin\theta_C + s\cos\theta_C \end{aligned}$$

The arbitrary angle is called the Cabibbo angle. It was introduced in 1963 as part of the $(V - A)$ theory of low-energy weak interactions. In leptonic decays, assuming a 4-fermion $(V - A)$ interaction, the interaction can be written as

$$\frac{G_F}{\sqrt{2}} J^\mu_{\text{lep}}(x) J^\dagger_{\text{lep}\,\mu}$$

In hadronic decays there is a strangeness conserving part (e.g. $n \to pe^-\nu$) and a strangeness changing part (e.g. $K \to \mu\nu$, $K \to \pi\mu\nu$). The strengths of the 2 parts are not the same, so Cabibbo proposed that in semileptonic decays one writes the interaction as:

$$\frac{G_F}{\sqrt{2}} \left[\cos\theta_C J^\mu_{\Delta S=0}(x) + \sin\theta_C J^\mu_{\Delta S=1}(x)\right] J^\dagger_{\mathrm{lep}\,\mu} + \text{Hermitian conjugate}$$

One can determine the cosine of the Cabibbo angle by measuring beta decays in $0^+ \to 0^+$ transitions in which the nuclei belong to the same isospin multiplet and comparing them with G_F measured in muon decay. These measurements yielded $\cos\theta \approx 0.970 - 0.977$, so $\theta_C \approx 13°$. The values of $\sin\theta_C$ derived from $\Delta S = 1$ decays are consistent with this value. For example one can measure the ratios:

$$\frac{\Gamma(K^+ \to \mu^+\nu_\mu)}{\Gamma(\pi^+ \to \mu^+\nu_\mu)} \propto \sin^2\theta_C$$
$$\frac{\Gamma(K^+ \to \pi^0 e^+\nu_e)}{\Gamma(\pi^+ \to \pi^0 e^+\nu_e)} \propto \sin^2\theta_C$$

Correcting for kinematic factors one finds that the $\Delta S = 1$ transitions are suppressed by a factor of about 20, which implies that $\sin\theta_C \approx 0.22$, or $\theta_C = 12.7°$.

Within this framework one can have so-called Cabibbo favoured (or allowed) transitions and Cabibbo suppressed transitions. Viewed in a more modern perspective this implies the following corrections to the vertices of the relevant Feynman graphs:

⇒ *Slide: Cabibbo favoured and suppressed transitions (cabibbo.mnf)*

Writing down the matrix element for weak interactions involving quarks in terms of quark currents:

$$\mathcal{M} = \frac{4G_F}{\sqrt{2}} J^\mu J^\dagger_\mu$$

where

$$J^\mu = \begin{pmatrix} \bar{u} & \bar{c} \end{pmatrix} \frac{\gamma^\mu(1-\gamma^5)}{2} V \begin{pmatrix} d \\ s \end{pmatrix}$$

and

$$V = \begin{pmatrix} \cos\theta_C & +\sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{pmatrix}$$

************************* *Begin skip* *************************

Writing the same thing in terms of the interaction Lagrangian we have:

$$\mathcal{L}_{\mathrm{int}} = -\frac{g}{\sqrt{2}} \left[\bar{U}'_L \not{W} D'_L + \text{herm. conj.}\right]$$
$$= -\frac{g}{\sqrt{2}} \left[\bar{U}_L \not{W} V D_L + \text{herm. conj.}\right]$$

with the following definitions

$$\psi_L = \frac{1}{2}(1-\gamma^5)\psi$$
$$\not{W} = \gamma_\mu W^{\mu +}$$
$$\frac{g^2}{8M_W^2} = \frac{G_F}{\sqrt{2}}$$

ψ_L selects the left-handed component of the lepton wavefunction. Is this clear?

The particle multiplets are defined as:

$$U' = \begin{bmatrix} u' \\ c' \end{bmatrix} \quad D' = \begin{bmatrix} d' \\ s' \end{bmatrix}$$

$$U = \begin{bmatrix} u \\ c \end{bmatrix} \quad D = \begin{bmatrix} d \\ s \end{bmatrix}$$

************************* *End skip* *************************

The matrix V is constructed to be unitary, i.e. $V^\dagger V = V^{-1}V = 1$, so that the interaction Lagrangian can be written in either of the forms given above. For the case of 2 generations the corresponding 2×2 matrix is given by:

$$\begin{pmatrix} \cos\theta_C & -\sin\theta_C \\ +\sin\theta_C & \cos\theta_C \end{pmatrix} \begin{pmatrix} \cos\theta_C & +\sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

************************* *Begin skip* *************************

1.6.2 Helicity states

Useful to see why one can write

$$\psi_L = \frac{1}{2}(1-\gamma^5)\psi$$

The Dirac equation in momentum space is given by

$$(\gamma^\mu p_\mu - m)u = 0$$

with

$$u = \begin{pmatrix} u_A \\ u_B \end{pmatrix}$$

where u_A, u_B are 2 component spinors which are related by:

$$u_A = \frac{1}{E-m}\,(\vec{p}\cdot\vec{\sigma})\,u_B$$
$$u_B = \frac{1}{E+m}\,(\vec{p}\cdot\vec{\sigma})\,u_A$$

One can also show that

$$\gamma^5 u(p) = \begin{pmatrix} \frac{\vec{p}\cdot\vec{\sigma}}{E+m} & 0 \\ 0 & \frac{\vec{p}\cdot\vec{\sigma}}{E-m} \end{pmatrix} u(p)$$

where γ^5 is the 4×4 matrix

$$\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

For a massless particle $E = |\vec{p}|$. Thus we have

$$\gamma^5 u(p) = (\hat{p}\cdot\vec{\Sigma})u(p)$$

where

$$\vec{\Sigma} = \begin{pmatrix} \vec{\sigma} & 0 \\ 0 & \vec{\sigma} \end{pmatrix}$$

and $\frac{1}{2}\vec{\Sigma}$ is then the spin matrix of a Dirac particle. $\hat{p}\cdot\vec{\Sigma}$ is the definition of the *helicity* of a particle and it has eigenvalues ± 1 for a massless particle. Inserting the explicit definition of γ^5 one can show that:

$$\begin{aligned} \frac{1}{2}(1-\gamma^5)u(p) &= 0 \qquad \text{if } u(p) \text{ has helicity } +1 \\ \frac{1}{2}(1-\gamma^5)u(p) &= u(p) \quad \text{if } u(p) \text{ has helicity } -1 \end{aligned}$$

i.e. $\frac{1}{2}(1-\gamma^5)$ projects out the left-handed (helicity -1) component of a spinor.

One can write down the left and right-handed states for both fermions and antifermions:

$$u_L(p) = \tfrac{(1-\gamma^5)}{2}u(p) \quad v_L(p) = \tfrac{(1+\gamma^5)}{2}v(p)$$

$$u_R(p) = \tfrac{(1+\gamma^5)}{2}u(p) \quad v_R(p) = \tfrac{(1-\gamma^5)}{2}v(p)$$

What about the adjoint spinors?

$$\begin{aligned} \bar{u}_L(p) &= u_L^\dagger(p)\gamma^0 \\ &= u^\dagger \frac{1-\gamma^5}{2}\gamma^0 \\ &= u^\dagger\gamma^0\frac{1+\gamma^5}{2} \\ &= \bar{u}^\dagger\frac{1+\gamma^5}{2} \end{aligned}$$

where the second line uses the fact that γ^5 is a hermitian matrix. The definitions of $\bar{u}_R(p)$, $\bar{v}_L(p)$, $\bar{v}_R(p)$ are similar. These spinors are called "chiral" fermion states. One can write the electromagnetic and weak current in terms of these states:

$$\begin{aligned} (j_\mu)^{\mathrm{CC}} &= \bar{u}^\nu\gamma_\mu(1-\gamma^5)u^e \\ &= \bar{u}^\nu\gamma_\mu\left[\frac{(1-\gamma^5)}{2}\right]^2 u^e \\ &= \bar{u}^\nu\frac{(1+\gamma^5)}{2}\gamma_\mu\frac{(1-\gamma^5)}{2}u^e \\ &= \bar{u}_L^\nu\gamma_\mu u_L^e \end{aligned}$$

i.e. the weak charged current couples left-handed states to left-handed states. For the electromagnetic current one can show that:

$$\begin{aligned}(j_\mu)^{\mathrm{EM}} &= \bar{u}^e \gamma_\mu u^e \\ &= \bar{u}^e_L \gamma_\mu u^e_L + \bar{u}^e_R \gamma_\mu u^e_R\end{aligned}$$

For completeness the weak neutral current is:

$$\begin{aligned}(j_\mu)^{\mathrm{NC}} &= \bar{u}^e \gamma_\mu (g^f_V - g^f_A)\gamma^5 u^e \\ &= \bar{u}^e_L \gamma_\mu (g^f_V + g^f_A) u^e_L + \bar{u}^e_R \gamma_\mu (g^f_V - g^f_A) u^e_R \\ &= g^f_L \bar{u}^e_L \gamma_\mu u^e_L + g^f_R \bar{u}^e_R \gamma_\mu u^e_R\end{aligned}$$

where:

$$\begin{aligned}g^f_L &= g^f_V + g^f_A \\ g^f_R &= g^f_V - g^f_A\end{aligned}$$

We see from these considerations that vector and axial vector currents both conserve helicity.

************************* *End skip* *************************

End of Lecture 01

1.6.3 CKM matrix construction

If we extend the above to 3 generations V becomes a 3×3 unitary matrix. Kobayashi and Maskawa extended the matrix to a 3×3 matrix in 1973 – before the charm quark was even discovered! As we will see a 3×3 unitary matrix contains one complex parameter that can be used to explain CP violation within the framework of the Standard Model. How many free parameters are there in a unitary matrix?

An $n \times n$ complex matrix contains $2n^2$ real and complex parameters. Unitarity imposes n^2 constraints. We have a total of $2n$ quarks, the phases of which can be changed independently. This implies that there are $2n$ fewer parameters. However, we have to fix one of the phases, as otherwise the matrix would be unchanged. We thus have $n^2 - (2n - 1)$ free parameters.

A real (unitary) matrix has $n(n-1)/2$ free parameters, as this is the number of independent rotations in n dimensions. Thus a unitary matrix has

$$n^2 - (2n-1) - \frac{n(n-1)}{2} = \frac{(n-1)(n-2)}{2}$$

imaginary parameters. We therefore have:

n = 2	1 real	0 imaginary
n = 3	3 real	1 imaginary

parameters. In the case of $n = 3$ we therefore have 4 parameters which are fundamental *unknown* parameters in the Standard Model that have to be determined by experiment. If they can be determined in several different ways, comparisons of the measurements test the validity of the Standard Model.

With 3 generations we can write the matrix as:

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

It gives the coupling of the W^+ boson to $\bar{u}_i d_j$ quark pairs. There are many possible parametrisations of a 3×3 unitary matrix. I will use the form recommended by the Particle Data Group, which differs somewhat from the form originally proposed by Kobayashi and Maskawa in 1973.

$\Rightarrow$ *Slide: The CKM Matrix (ckm.tex)*

The advantages of this form are that the rotation angles are defined and labelled in a way that relates the mixing of two specific generations. If one of the angles vanishes so does the mixing between these two generations. In the limit $\theta_{23} = \theta_{13} = 0$ the third generation decouples and we recover the usual Cabibbo mixing of the first two generation with θ_{12} identified as the Cabibbo angle.

The matrix elements in the first row and third column have a simple form and have all been measured in decay processes. c_{13} is known to differ from unity only in the 6th decimal place. Thus to a very good approximation:

$$\begin{aligned} V_{ud} &= c_{12} \\ V_{us} &= s_{12} \\ V_{ub} &= s_{13} e^{-i\delta_{13}} \\ V_{cb} &= s_{23} \\ V_{tb} &= c_{23} \end{aligned}$$

The phase δ_{13} lies in the range $0 \leq \delta_{13} \leq 2\pi$. Non-zero values generally break CP invariance for the weak interactions, as will be discussed later.

A useful approximate form of the CKM matrix was introduced by Wolfenstein in 1983. One can choose the quark phases so that the diagonal and $(n-1)$ above the diagonal elements are real. As we will see the angles θ_{ij} are small and V_{cs} is almost real, so only small changes to the quark phases are necessary. The diagonal elements are close to 1 and the dominant off-diagonal element is $V_{us} \approx -V_{cd} \approx \sin\theta_C \equiv \lambda = 0.22$. Thus to order λ^2 we have:

$$V = \begin{pmatrix} 1-\lambda^2/2 & \lambda & . \\ -\lambda & 1-\lambda^2/2 & . \\ . & . & 1 \end{pmatrix}$$

As we will see $V_{cb} \approx 0.04$. We can therefore set $V_{cb} = A\lambda^2$, where A is of the order of 1. If V_{td} and V_{ub} are small then unitarity requires

$$\begin{aligned} V_{ts} &\approx -A\lambda^2 \\ V_{ub} &\approx A\lambda^3 \times \mathcal{O}(1) \end{aligned}$$

We have to introduce the phase somewhere though, so we write:

$$V_{ub} \approx A\lambda^3(\rho - i\eta)$$

Applying the unitarity requirement we then have:

$$V_{td} \approx A\lambda^3(1 - \rho - i\eta)$$

The full parametrisation is shown in the slide.

⇒ *Slide: Wolfenstein parametrisation of the CKM matrix (ckm.tex)*

A, ρ and η are real numbers that were intended to be of order unity. Must stress that this is an approximate form that is accurate to order λ^3. The matrix to oder λ^5 can be found in more detailed papers (e.g. R.V. Kowalewski `hep-ex/0305024`). The physics cannot depend on such parametrisations. One just has to make sure that a single one is used consistently.

Note that it is pure convention to say that d, s, b quarks are mixed. One could follow a completely analogous procedure for u, c, t.

1.6.4 MNS matrix construction

The construction of the MNS matrix follows along almost identical lines. The matrix relates the weak eigenstates, called e, μ, τ with the mass eigenstates $1, 2, 3$. The PDG form shown on Slide 7 is just as valid for the neutrino sector. However, the hierarchy of the parameters is very different, so the matrix written in a factorised form is often a more convenient representation.

⇒ *Slide: Factorised MNS matrix (ckm.tex)*

To a reasonable approximation, taking the measured values, the matrix can be written in the following form (ignoring the imaginary component):

⇒ *Slide: Tribimaximal mixing for MNS matrix (ckm.tex)*

According to Wikipedia: the name tribimaximal reflects the commonality of the tribimaximal mixing matrix with two previously proposed specific forms for the P-MNS matrix, the trimaximal and bimaximal mixing schemes, both now ruled out by data. In tribimaximal mixing, the ν_2 neutrino mass eigenstate is said to be "trimaximally mixed" in that it consists of a uniform admixture of ν_e, ν_μ, ν_τ flavour eigenstates, i.e. maximal mixing among all three flavour states. The ν_3 neutrino mass eigenstate, on the other hand, is "bimaximally mixed" in that it comprises a uniform admixture of only two flavour components, i.e. ν_μ and ν_τ maximal mixing, with effective decoupling of the ν_e from the ν_3, just as in the original bimaximal scheme.

As you can see the values are very different form those of the quarks. This is an observation that has not yet been explained.