I. INTRODUCTION

One of the most fundamental questions in Astrophysics and Cosmology is the nature of the Dark Matter (hereafter DM) which pervades the universe. At least 90% of the mass of the Universe does not emit or absorb electromagnetic radiation in appreciable amounts and is inferred only through its gravitational effects. Its nature is unclear, but the hypothesis that the DM is made out of ordinary baryons encounter difficulties; primordial nucleosynthesis along with the predilection for a Universe of critical density seem to preclude it, popular models of galaxy formation favor a large non-baryonic component, and allowable forms such as Jupiter size objects or large stellar remnants are thought not to form in the required numbers. Among the non-baryonic possibilities (primordial black holes, exotic objects, elementary particles, etc.) the idea that the DM could consist of the lowest mass (stable) member of a yet unknown family of elementary particles is fairly attractive. Many current improvements of the Standard Model of particle physics predict such a family, the most familar example being supersymmetry.

It has been suggested that this hypothesis can be tested experimentally[1,2] and many groups are upgrading existing experiments or developing new experiments to directly detect dark matter particles. In view of this experimental effort, it is important to assess the real model independence of rate predictions and constraints from accelerator experiments. This is what we attempt to do in this paper for a wide class of models where the dark matter is assumed to consist of massive particles. We will call these particles δ's where δ could stand for a neutrino, photino, etc..

The class of experiments we will be discussing consists of low-temperature, kilogram size detectors designed to measure the small (order keV) recoil energy deposited when a DM particle moving through the halo of the galaxy scatters elastically off a nucleus in the detector. The rate in such a detector depends upon several halo parameters, the mass of the δ, the mass of the detector nucleus, and most importantly upon the $\delta - N$ elastic scattering cross section. In Section II we discuss this interaction rate and the uncertainties in it.

Making the fundamental assumption that the δ particles were once in thermodynamic equilibrium with quarks and leptons, presumably through "annihilation" channels such as $\delta\bar{\delta} \leftrightarrow q\bar{q}, e^- e^+$, etc., implies that the current relic density of DM particles is related to the annihilation cross section at the time of "freeze-out". In Section III we analyze this argument.

By "crossing symmetry", as indicated in Section IV, the annihilation cross section can be related to the elastic cross section mentioned above, leading in a rather model independent way to a lower limit for the interaction rate of δ particles from the galaxy with a target in the laboratory. The degree of model independence of this result, along with other uncertainties are discussed in Section IV. Finally, in Section V we consider another possible "crossing", that is, we show how the results of high energy production experiments such as ASP can constrain DM detection experiments and influence detector design strategies. In particular, accelerator experiments in general rule out lighter δ's, although once again, we point out how this is model dependent conclusion. Section VI summarizes our main conclusions.

II. THE RATE IN THE DETECTOR

The number of events per kilogram per day in a DM detector can be written[1,3,4]

$$R = \sqrt{\frac{8}{3\pi}} \eta_t \eta_v \eta_c \eta_e \frac{\langle v \rangle_{halo} \rho_{halo} \sigma_{el}}{m_N m_\delta}, \tag{1}$$

where $\rho_{halo} \sim .3$ GeV cm^{-3} is the local density of DM particles, $\langle v \rangle_{halo} \sim 270$ km/sec is the average speed of a particle in the halo, m_N is the mass of the detector nucleus, m_δ is the mass of the DM particle, and σ_{el} is the elastic cross section. The η factors[3,4] describe various modifications to the rate and are generally of order unity. The $\eta_v \sim 1.3$ factor is the enhancement due to the motion of the Sun and Earth through the halo, $\eta_c \leq 1$ is a coherence loss factor, more properly contained in σ_{el}, which is small for heavy detector nuclei and heavy δ's. The $\eta_t \leq 1$ factor accounts for the detector threshold and is small for high threshold devices and low energy depositions. Finally $\eta_e \sim 1$ takes into account a possible escape velocity for the galaxy. See Refs. 3 and 4 for details.

Eq. (1) was derived using an isothermal potential model of the galaxy's halo as described in Refs. 5, 6, and 3. Since ρ_{halo} and $\langle v \rangle_{halo}$ depend crucially upon both the form and parameter values of this model and neither are well determined we have a factor of two or more uncertainty introduced into the rate at this point. The halo is expected to be non-isothermal,[7] but by far the largest uncertainty comes from the determination of ρ_{halo}.[5] In comparison, the existence of a galactic escape velocity and the effect of a possible halo rotation are small. The mass of the δ is unknown, but $m_\delta > 1$ GeV are of interest. The mass of the detector nucleus is under the control of the experimenter and using detectors with different m_N may help discriminate signal from background. The biggest uncertainty in eq. (1) is σ_{el}, which in principle could be zero. As we will show, however, some expectations of σ_{el} do exist.

In conclusion, we see that a great deal of uncertainty in the rate comes simply from the relatively unknown nature of the galactic halo. If DM particles are found and their properties measured (perhaps with help from accelerator experiments) then useful information about the galactic halo could be extracted; information which may be unobtainable in any other way.

III. CURRENT DENSITY AND ANNIHILATION RATES

The hypothesis that the dark matter is made out of particles δ, which were once in thermal equilibrium with ordinary particles, limits considerably the freedom of the particle physics model. The present density of δ's in the universe is a function of their annihilation rate at the time they dropped out of equilibrium (the "freeze out" time). The argument worked out in detail for neutrinos by Lee and Weinberg[8] among others is simple. At early times, the temperature and densities are high enough so that reactions such as $\delta\bar{\delta} \leftrightarrow q\bar{q}$, $\delta\bar{\delta} \leftrightarrow e^-e^+$, $\delta\bar{\delta} \leftrightarrow \nu\bar{\nu}$, etc. can maintain kinetic and chemical equilibrium. As the universe expands and the temperature drops below the mass of the δ particle, its equilibrium number density drops exponentially due to the Boltzmann factor, $e^{-m_\delta/T}$. However, at a certain temperature, determined by the annihilation cross section, the number density of δ's drops too low for annihilation to proceed in equilibrium; the δ's "freeze-out", the number density cannot change appreciably and we are left with a well determined abundance today. Therefore, in the case of no initial asymmetry, the present density of δ's ***fixes a normalization point to the interaction strength of the model considered.*** Note that these considerations are extremely general, but do not apply to particles such as axions[9] which according to current schemes were never in equilibrium with the rest of matter.

Several approximation schemes to solve the Boltzmann equation have been developed[10,11] for the case where the annihilation rate can be written

$$\langle \sigma v \rangle_{\mathrm{ann}} = a + b \langle v^2 \rangle \,, \tag{2}$$

where $\langle v^2 \rangle = 6kT/m_\delta$ is the thermally averaged relative velocity of the particle and anti-particle. The two terms corresponds to s-wave and p-wave annihilation which are dominant for the low energy at freeze-out. In our numerical estimation below, we use the method of ref. 11. Fig. 1 gives the annihilation cross section for two representative models, the massive Dirac "neutrino" (Fig. 1a, $\alpha = 0$ curves) where the s-wave term is dominant, and a pure photino model with degenerate scalar fermions (Fig. 1b) where the p-wave term is important. Relevant formulae are given in Appendix A. In these calculations we have allowed the "coupling strengths" (i.e. the hypercharge for neutrinos or the scalar fermion masses for photinos) to adjust themselves in order to provide a given density today. The curves are labelled with the relevant $\Omega_\delta h^2$, where $\Omega_\delta = \rho_\delta / \rho_{\mathrm{crit}}$ is the present ratio of the average δ density in the universe to the critical density ($\rho_{\mathrm{crit}} = 1.05 \times 10^{-5} h^2$ GeV cm^{-3}), and h is the Hubble constant in units of 100 km s^{-1} Mpc^{-1}. In Fig. 1a we have also drawn the expected annihilation cross section for a Standard Model neutrino (Ω is allowed to vary). It is seen that after some oscillation at low mass due to the various thresholds ($\tau\bar{\tau}$, $c\bar{c}$, $b\bar{b}$) above a few GeV

$$\langle \sigma v \rangle_{ann} \sim \frac{10^{-26} \mathrm{cm}^3 \mathrm{sec}^{-1}}{\Omega_\delta h^2}, \tag{3}$$

where the exact value of the coefficient depends on the dominance of s-wave or p-wave annihilation. It is a striking coincidence and perhaps meaningful that the cross sections needed to supply a critical density are in the range of the weak interactions.

In the last paragraph, we took the attitude of determining the interaction strength through the current density of dark matter, that is, fixing the unknown parameters of the model using Ω. However, in some cases, such as massive neutrinos, this is unnatural because the model is essentially fixed. In this case two situations have to be considered. It could be that the considered particle is only a minority component of the dark matter. Fig. 2 shows the approximate Ωh^2 obtained for typical models. In this case, when looking for elastic interaction of the considered dark matter particles, the particle density in the halo ρ_{halo} has to be reduced proportionally (see eq. (1))

$$\rho_{halo} \rightarrow \rho_{halo} \frac{\Omega_\delta}{\Omega_{\rm DM}}, \tag{4}$$

where $\Omega_{\rm DM}$ is the ratio of the total dark matter density to the critical density. This assumes that no segregation between the various types of dark matter has occurred.

Another scenario could be that of an initial asymmetry.[11,12] If, for instance, the number of δ particles is larger than the number of $\bar{\delta}$'s, a large annihilation rate may be compatible with the current density: even if all $\delta\bar{\delta}$ pairs disappear, one is left with the initial excess of δ's. This is, of course, what happened for protons in the Universe. Fig. 1a gives an example for Dirac "neutrinos" of the effect of an initial asymmetry. The variable α is defined as the (invariant) ratio of the excess number density to the entropy density, s

$$\alpha = \frac{n_\delta - n_{\bar{\delta}}}{2s} \tag{5}$$

where $s \simeq 7 n_{\rm photon}$ and $n_{\rm photon}$ is the number density of photons. Note that for baryons $\alpha_b \sim 2 - 7 \times 10^{-11}$ (ref. 13). Fig. 1a shows, as already noted by several authors, that a heavy neutrino of mass 10 GeV and an asymmetry similar to that of baryons will nicely lead to $\Omega_\delta \sim 1$, which is at least a remarkable coincidence.

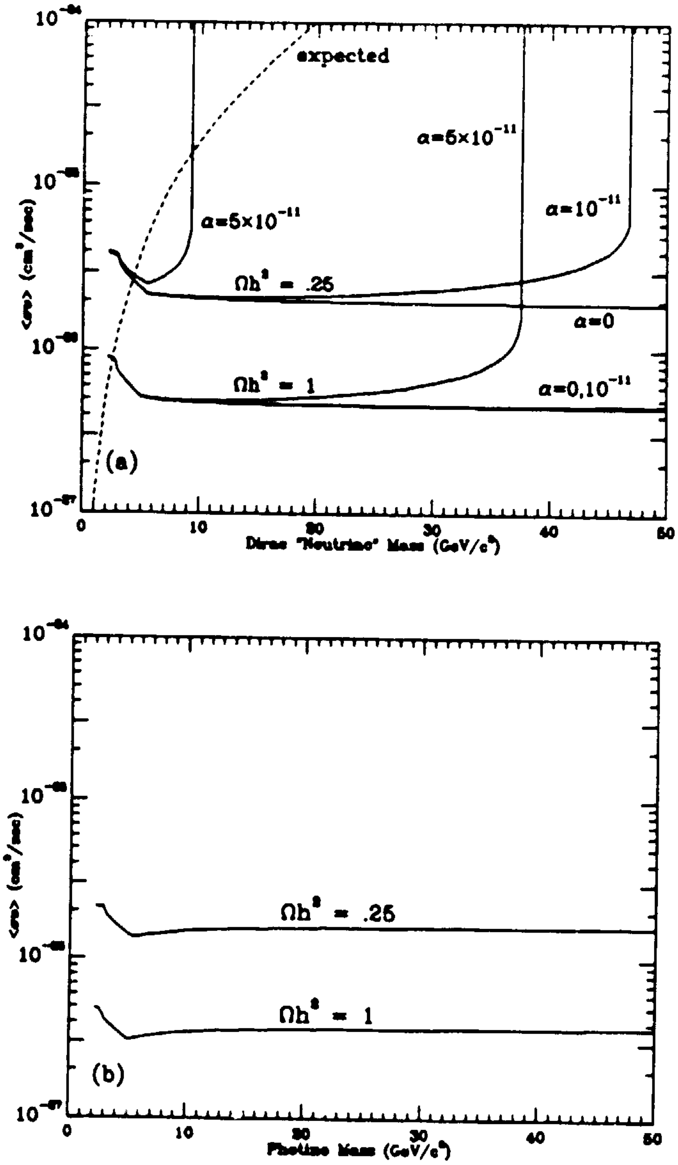

Fig. 1. Annihilation cross sections required to give $\Omega = 1$. A "Dirac neutrino" (a) and a pure photino (b) are shown for two values of the Hubble parameter, h. The the hypercharge and squark mass respectively are adjusted to give the stated value of Ωh^2. Several values of asymmetry (α), are shown for the "Dirac neutrino", as is the cross section expected for a standard model hypercharge (dashed line).

But note that for a given asymmetry there are either no or two masses which satisfy the current density constraint. For instance if we impose $\alpha_\delta = \alpha_b$ and $\Omega_\delta + \Omega_b = 1$, the upper mass solution is $(m_b + m_\delta) \sim h^2/(\alpha s)$, *independently* of the annihilation cross section if it is large enough (which is the case for heavy Dirac neutrinos). This is seen in Fig. 1a where the line labeled "expected", which shows $\langle \sigma v \rangle$ as calculated from the Standard Model, intersects the values of $\langle \sigma v \rangle$ needed to give the labeled $\Omega_\delta h^2$ in two places. This model has some attractiveness with respect to a model without asymmetry. In the latter case, the fact that we live in a universe which is matter dominated is the result of a very peculiar value of the annihilation cross section which may appear as an unlikely conspiracy. A nice feature from the experimental point of view is that requiring the δ's to make up the DM implies a *lower* limit on the annihilation cross section, even if an initial asymmetry is allowed. Massive particles which interact too weakly with ordinary matter would over-close the universe ($\Omega_\delta \gg 1$) and are therefore ruled out.

The above argument is fairly general, but we list here some possible loopholes. First, as mentioned, the "Lee-Weinberg" argument does not apply for hypothetical particles such as the axion[9] which according to current schemes were never in thermal equilibrium. Such particles give rise to perhaps the most attractive alternative to the WIMP (weakly interacting massive particle) scenarios described here, but will not be discussed further. If the δ's have a long lifetime, but are not absolutely stable, then their density may have been partially diluted by decay, and in addition their decay products may constitute some of the DM. In this case the above argument will over-estimate the interaction strength. However, such scenarios encounter significant difficulties and are not very attractive. There is also uncertainty introduced into the relic abundance calcuations due to uncertainties concerning the quark-hadron phase transition. Finally, since in equilibrium, the number density of δ's does not exceed the number density of photons, very light or massless δ's can exist with low interaction strengths and evade the universe over-closure argument. Neutrinos or photinos with masses under 100 eV are examples. These very light particles also deposit so little energy in a detector that no feasible means of detection has yet been suggested. However, since they would have been relativistic at the time galaxies could first have started to

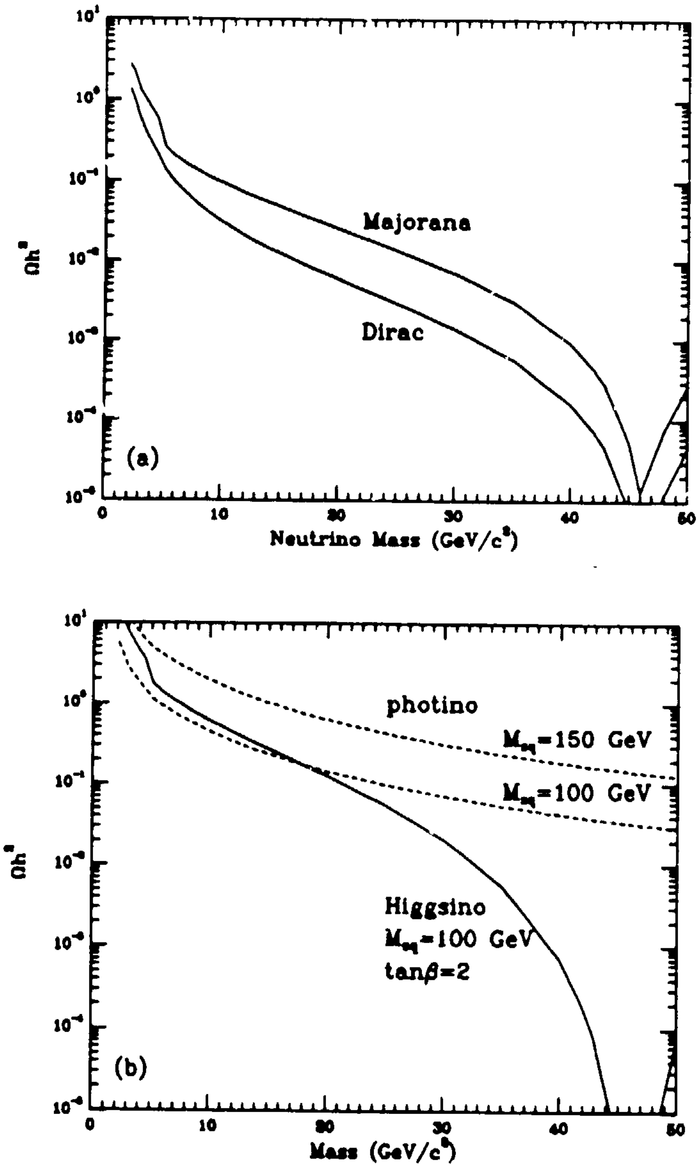

Fig. 2. Approximate relic abundances of Dirac and Majorana neutrinos, pure photinos and Higgsinos as a function of their mass.

form, they would constitute "hot" dark matter – which is currently not favored in galaxy formation scenarios, and they would also be unable to cluster in the observ·u halos of small galaxies, requiring the existence of at least two types of dark matter.

IV. CROSSING SYMMETRY

So far we have established that under fairly general conditions, the present density of dark matter particles gives a lower limit to their interaction strength. How can we transform this result into a lower limit on the interaction rate of particles from the galactic halo with a suitable target in the laboratory? The basic result that the elastic cross section follows from the annihilation cross section via crossing symmetry does imply a lower limit on the interaction rate in a detector, however, as we will describe, there are several important loopholes and caveats which must be addressed.

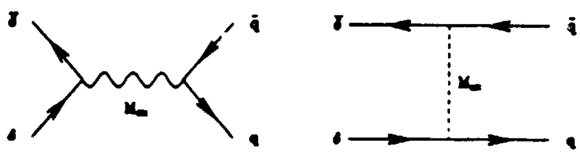

Fig. 3. Feynman diagrams for a generic interaction between quarks (q) and dark matter particles (δ).

Fig. 3 shows examples of two possible interactions which, if read from left to right, describe annihilation and if read from bottom to top describe elastic scattering. The crossing symmetry which allows an elastic matrix element to be written in terms of an annihilation matrix element is the basis for suggesting that a lower limit on the annihilation cross section can be translated into a lower limit on the scattering cross section and therefore on the interaction rate in a detector.

Ignoring propagator momenta for now, a generic annihilation cross section resulting from diagrams such as Fig. 3 can be written

$$\sigma_{ann} v \simeq \sum_{f<\delta} \frac{g^4 m_\delta^2 C_\delta^2 C_f^2}{M_{ex}^4} \sqrt{1 - m_f^2/m_\delta^2} \left(a_f + b_f \frac{m_f^2}{m_\delta^2} + c_f v^2 \right) , \qquad (6)$$

where M_{ex} is the mass of the exchanged particle, gC_δ is the generic coupling of the δ to the exchange particle, and gC_f is the coupling of the fermionic annihilation product (quark or lepton) to the exchange particle. The model dependent factors a_f, b_f and c_f are of order unity, v is the relative velocity of the δ's, and the sum is over all fermions with mass less than the δ. This is not the most general formula, but is general enough to show most of the complexities we will consider. Exceptions include scalar decay products (e.g. Higgs'), exchanged fermions ($\sigma_{ann} \propto M_{ex}^{-2}$) and the possibility of additional exchange particles with the resulting interference.

An important consideration is the type of coupling the δ has with the fermion. As examples we will consider a spin one-half δ particle with a vector-vector coupling ($\bar{\delta}\gamma_\mu\delta\bar{f}\gamma^\mu f$), axial vector-axial vector coupling ($\bar{\delta}\gamma_\mu\gamma_5\delta\bar{f}\gamma^\mu\gamma_5 f$), scalar-scalar coupling ($\bar{\delta}\delta\bar{f}f$), and pseudoscalar-pseudoscalar coupling ($\bar{\delta}\gamma_5\delta\bar{f}\gamma_5 f$). Although mixed and tensor couplings are possible, these four serve to illustrate our points and span the set of dark matter candidate particles usually considered.

For particles with vector-vector couplings (such as Dirac neutrinos) all the coefficients a_f, b_f, and c_f are typically present, while for particles with axial vector-axial vector couplings, $a_f = 0$. This is the well known "p-wave" suppression which exists for majorana particles such as photinos and majorana neutrinos. Note that since at freeze-out $\langle v^2 \rangle \approx 1/3$ this is not necessarily a big suppression. Scalar-scalar interactions have $a_f = b_f = 0$, while pseudoscalar-pseudoscalar interactions have $b_f = 0$. In these last two cases $C_\delta C_f$ usually contains a Yukawa coupling factor of m_f/m_δ which suppresses σ_{ann} for $f = u, d, e, \mu$, and ν, but can contribute for b, c, or τ near threshold. Finally note that a t-channel scalar exchange (see Fig. 3) can be written as a sum of all four types of couplings and so typically contains all three terms displayed in eq. (6).

Using Fig. 3, a generic elastic scattering cross section for use in eq. (1) can be written

$$\sigma_{el} \simeq \frac{g^4 m_\delta{}^2 m_N^2 C^2}{M_{ex}^4 (m_\delta + m_N)^2}, \tag{7}$$

where m_N is the mass of the detector nucleus and C^2 is a coherence factor which hides a great deal of physics. Before discussing C^2 we can blindly use eq. (6), eq. (7) and eq. (3) to find

$$\sigma_{el} \geq \frac{m_N^2}{(m_\delta + m_N)^2} \left(\frac{10^{-37}\mathrm{cm}^2}{\Omega_\delta h^2} \right) \left[\frac{C^2}{\sum C_f^2 C_\delta^2 (a_f + b_f \langle v^2 \rangle + c_f m_f^2 / m_\delta{}^2)} \right]. \tag{8}$$

Taking the factor in square brackets to be of order unity we see that the elastic scattering cross section is therefore generically of weak strength or larger. Using this in eq. (1) we find for m_δ of order 10 GeV

$$R \gtrsim 1 \text{ event kg}^{-1} \text{ day}^{-1} \tag{9}$$

which is a quite substantial rate, probably within reach of the current and next generation of DM detectors. This encouraging result has given rise to the large effort by experimenters – but it is worth going back and carefully examining the simplifications and assumptions that went into it.

First note that we equated M_{ex} in eq. (6) with M_{ex} in eq. (7). For Z^0 exchange this is certainly valid, but for squark/selectron exchange as presumed for photino dark matter this identity need not hold. If, for example, the selectron is much lighter than the up and down squark, then relic abundance will be determined by the selectron exchange annihilation $\tilde{\gamma}\tilde{\gamma} \rightarrow e^- e^+$, while the elastic cross section must proceed via the smaller squark exchange. For many supersymmetric models $M_{selectron} < M_{squark} < 3M_{selectron}$, so we expect a suppression in eq. (9)by a factor of $(M_{selectron}/M_{squark})^4$ which varies between one and 1/81 in a model dependent way.

The main model dependencies in eqs. (6) and (7) are contained in C^2 and in the sum over annihilation products. To find C^2 we must consider vector, axial vector, scalar, etc. interactions separately. First consider a particle with axial vector-axial vector coupling. Here

$$C^2 \propto |\sum_q \langle N|A_q \bar{q}\gamma_\mu\gamma_5 q|N\rangle|^2,$$

where $\langle N|$ is the nucleus wave function, presumably made up of a sum of proton $\langle p|$ and neutron $\langle n|$ wave functions, which are presumably in turn made of sums of quark wave functions. The factor A_q is coupling constant of order unity. Knowledge of parton model physics, especially the proton and neutron structure functions is necessary in going from the quarks to the nucleons, while knowledge of nuclear physics is needed to sum up the nucleons' contribution.

The first step in the axial vector case is to note[1] that in the extreme non-relativistic limit relevant here, $\bar{q}\gamma_\mu\gamma_5 q$ is proportional to the spin. So we can define[1,14,15]

$$\langle p|\bar{q}\gamma_\mu\gamma_5 q|p\rangle = 2\Delta q \vec{s}_q,$$

where $\vec{s}_q$ is the the spin of quark q, and Δq is the fraction of the proton spin carried by quark q. Data from neutron decay leads to the relation $\Delta u - \Delta d = 1.25$, while hyperon decay and a flavor SU(3) assumption leads to $\Delta u + \Delta d - 2\Delta s = .682$. Traditionally one sets $\Delta s = 0$ and solves for Δu and Δd

$$\Delta s = 0, \quad \Delta u \approx .966, \quad \Delta d \approx -.284. \tag{10}$$

However, recent results from the EMC group[14] on polarized muon scattering can be interpreted to give a third equation,

$$.21\Delta u + .053\Delta d + .053\Delta s = .114 + \left\{ \frac{.316\alpha_s \Delta g}{2\pi} \right\},$$

where the last term includes a possible contribution to the spin from gluons. Setting $\Delta g = 0$, as suggested by several lines of reasoning[16,17] the EMC solution is[14]

$$\Delta s \approx -.234 \quad \Delta u \approx .732, \quad \Delta d \approx -.518. \tag{11}$$

Unfortunately, the use of the EMC Δq's give quite different values of C^2 as compared to the use of the traditional quark model values. So another factor of two to four uncertainty is introduced into the detection rate for particles with axial vector couplings. Hopefully theoretical and experimental work in the near future will clear up the present confusion.

Next, adding the proton and neutron wave functions using the one-particle nuclear shell model one finds[1,18] $C^2 \propto \lambda^2 J(J+1)$, where J is the total spin of the nucleus, and λ^2 is a Clebsch-Gordan coefficient which depends upon the shell model parameterization. Typically $.1 \leq \lambda^2 \leq 1$. Unfortunately again the simple one-particle nuclear shell model used is not very accurate and another large uncertainty is introduced at this point.

It is very important to note that $C^2 = 0$ for nuclei without spin ($J = 0$). Many common elements: ^{16}O, ^{12}C, ^{4}He, ^{72}Ge, and ^{28}Si have $J = 0$, and therefore $R = 0$ for them. So we see that it is quite possible to have $\sigma_{el} = 0$, even though σ_{ann} is substantial. The conclusion is that to detect DM particles which have only axial vector couplings one must build a detector out of an element which has a spin.

As an example of a particle with only axial vector couplings we consider the pure photino (another example is the majorana neutrino). Fig. 4 shows the rates in detectors made of various materials for the EMC and quark model values of the spin-dependent structure functions. Since $m_{\tilde{\gamma}} > 2$ or 3 GeV from the ASP experiment (see Sec. V.) rates between .01 and 1 event per kilogram per day are to be expected for the favorable elements shown. Backgrounds for current experiments are in the 5 event per kilogram per day range, so no limits on pure photinos are yet available.

Now consider the case of a vector-vector coupling. The non-relativistic limit of $\bar{q}\gamma_\mu q$ is not a vector but just a number, so $C^2 \propto |\sum_q A_q|^2$, where A_q is the coupling to quarks and the sum is over all quarks in the nucleus. Once again,

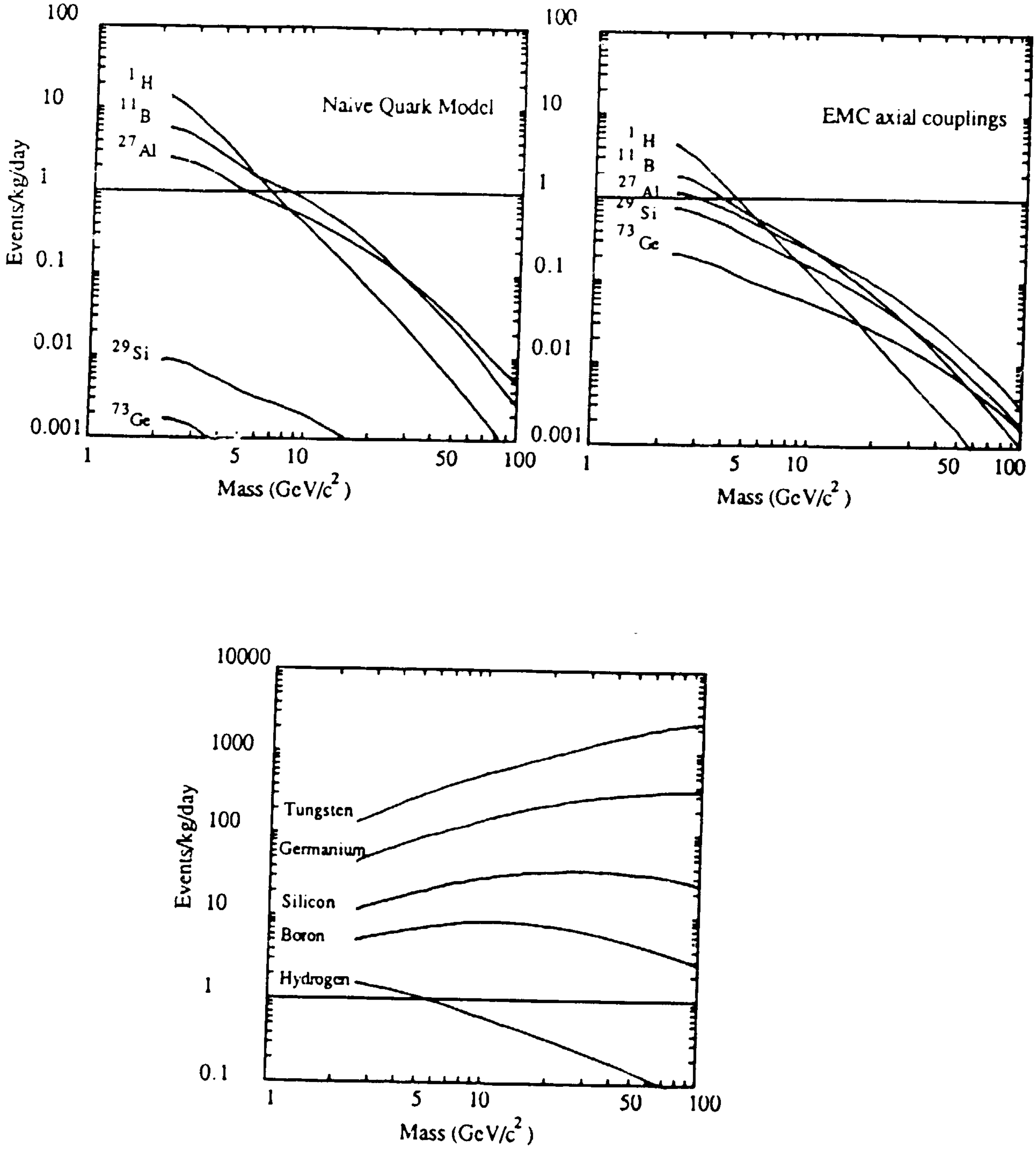

Fig. 4. Interaction rates of WIMPs (assumed to form the DM halo of our Galaxy) vs mass for various target materials for $\Omega_{DM} \approx 1$ from annihilation survival (*top panels*), and for $\Omega_{DM} \approx 1$ from particle-antiparticle asymmetry (*bottom panel*). In the top panels, the assumed WIMP is a pure photino, the sfermions are assumed to be degenerate with their mass determined so that $\Omega_{\tilde{\gamma}} h^2 \approx 0.25$. In the top left part of the figure, the axial couplings are determined from the naive quark model, while in the top right part they are as determined by EMC. In the bottom panel, the WIMP is a heavy Standard Model Dirac neutrino with $\Omega_\nu h^2 \approx 0.25$.

however, we may have swept uncertainty in the nuclear physics under the rug. An example of a particle with vector couplings is the Dirac neutrino, where $C^2 \propto |2\sum_q (T_{3L} - 2e_q \sin^2\theta_w)|^2$ which gives a factor of -1 for each neutron and a factor of $1 - 4\sin^2\theta_w \approx 0$ for each proton. So $C^2 \propto$ (number of neutrons)$^2 \approx$ 1600 for germanium. We see that in the case of vector-vector couplings very large enhancements in the rate are possible. Fig. 4 shows the expected rates for Dirac neutrino dark matter for a variety of elements. Because of the large predicted rates, existing experiments have already placed strong constraints on Dirac neutrino dark matter.[19] Dirac neutrinos with masses greater than around 10 GeV have been ruled out as the major component of the dark matter. A new silicon detector now operating should improve these limits considerably and either discover the dark matter or eliminate massive Dirac neutrinos as dark matter candidates. In conclusion, we see that detection of DM particles with vector couplings is very promising.

Now consider a particle with scalar-scalar couplings. Here $C^2 \propto |\sum_q \langle N|A_q \frac{m_q}{m_W}\bar{q}q|N\rangle|^2$, where we have explicitly included the Yukawa factor that usually accompanies a scalar coupling. This type of coupling is found when a Higgs boson is the exchange particle[20,21] or in supersymmetry when the generic lightest particle (neutralino) is considered[22,4] There are a few subtleties. In general m_q/m_W is very small for up and down quarks and one might think that this term could be ignored. But using techniques of Vainshtein, Zakharov and Shifman[23,20,4] one find $C^2 \propto m_N^2/m_W^2$. This is not a large enhancement, but can result in rates between 0.1 and 10 events per kilogram per day depending upon the mass of the detector nucleus. An interesting possibility is the case of light Higgs exchange. Here, if Z^0 exchange is also allowed, the annihilation via the light Higgs exchange may be suppressed due to the m_q/m_W Yukawa coupling (no coherence in annihilation channels) so $\sigma_{ann} \propto M_Z^{-4}$ as usual, while the coherence in the scattering cross section can be unsuppressed ($\sigma_{el} \propto M_{Higgs}^{-4}$) giving an overall enhancement in the detection rate of $(M_Z/M_{Higgs})^4$. Since Higgs bosons as light as 5 GeV are possible in the Standard Model we see that very substantial rates are possible. In fact, in the case of supersymmetry, current DM detectors are actually competing with accelerator experiments in the search for supersymmetry plus a light

Higgs.[21]

Now consider a pseudoscalar-pseudoscalar coupling. Here the non-relativistic limit of $\bar{q}\gamma_5 q$ gives an extra factor of v^2 in the elastic cross section, which is about 10^{-6} for scattering from the halo, and unfortunately makes the detection rate negligible. Once again this is a case where a substantial annihilation cross section exists, but the elastic cross section is small. Another case where a substantial annihilation cross section can exist and the elastic cross section vanish is the case of vector-axial vector mixed coupling. No candidate particles of these two types have yet been proposed, however.

Having illustrated the large model dependence contained in the coherence factor we should mention the model dependence in the annihilation sum. As pointed out previously, different exchange particles for different annihilation channels can lead to a model dependence, but alternatively the couplings to up and down quarks can be suppressed. Since mainly elastic scattering off up and down quarks is of interest, only the crossing of the up and down quark channels really matter. While an up and down quark suppression is not usual, one would like

$$\zeta = \frac{\sigma v(\bar{\delta}\delta \to \bar{u}u, \bar{d}d)}{\sigma v(\text{total})}$$

to be at least the canonical value of around 5% for the crossing argument to work. For most candidate particles $\zeta \geq 5\%$ and in fact the crossing does work, but the value of ζ is model dependent and typically a factor of $\zeta/5\%$ multiplies the final rate.

V. HIGH ENERGY PRODUCTION

Finally, we briefly mention another "crossing" possibility, that of extrapolating results from high energy δ production to annihilation in the early universe. As an example we consider the ASP experiment[24,25] which placed limits on the process $e^+e^- \to \gamma +$ "missing", where "missing" could be $\bar{\delta}\delta$. In general, $e^+e^- \to \gamma\bar{\delta}\delta$ is related to $e^+e^- \to \bar{\delta}\delta$, which is related to $\bar{\delta}\delta \to e^+e^-$. Detailed cross sections are shown in the appendix. In Fig. 5 for the case of a pure photino we show the ASP limit constraint on the annihilation cross section, where we have characterized the annihilation cross section by the mass of the exchanged selectron. Limits

$$m_{\tilde{\gamma}} > 2.1 \text{ GeV} \quad \text{if } \Omega_{\delta} h^2 = 1$$
$$m_{\tilde{\gamma}} > 4.3 \text{ GeV} \quad \text{if } \Omega_{\delta} h^2 = .25 \qquad (12)$$

are found at the 90% confidence level. Note that mass splitting of the scalar fermions ($M_{squark} = 3M_{selectron}$) increases the limits.

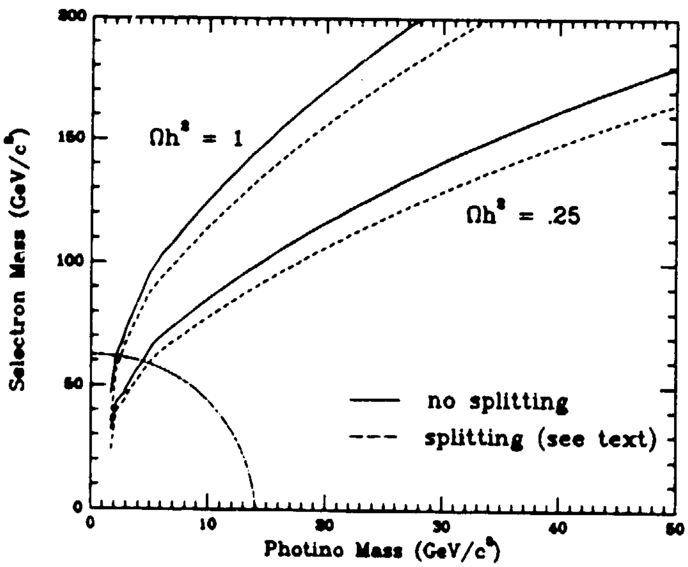

Fig. 5. Constraints on photino dark matter from the ASP experiment. Solid and dashed lines show the selectron mass required to give $\Omega = 1$, while the area below the dotted line is ruled out by ASP.

This kind of constraint is quite general, but again loopholes are possible. Annihilation can be through any of many channels – quark or lepton, while an ASP type experiment constrains only the coupling to the electron. For example, very little can be said about higgsinos, where the coupling to electrons is small and annihilation is mainly into heavier fermions. However, Fig. 6 shows the kind of constraint which could be found for higgsinos. A future high energy experiment

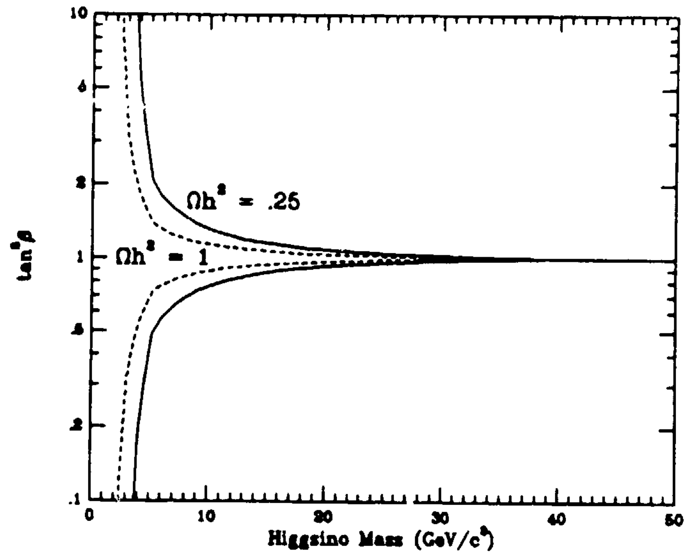

Fig. 6. Constraint on Higgsino dark matter using the ratio of Higgs expectation values, $\tan^2\beta$. The lines indicate the value of $\tan^2\beta$ required to give $\Omega = 1$.

which limits $\tan\beta$, the ratio of the Higgs vacuum expectation values, would limit the allowed range of DM higgsino mass. In general, however, we expect the combination of a lower limit on $\langle\sigma v\rangle_{ann}$ and an upper limit given by ASP to lead to a lower limit on m_{δ} in the GeV range. This is important, since negative results from accelerator experiments will gradually push up the allowed mass of DM candidates. Since low mass DM particles require low detector thresholds, this suggests that low thresholds may not be as important as the lower backgrounds needed to detect heavier DM particles. We see that the results of accelerator experiments can influence the design of DM detection experiments.

VI. CONCLUSIONS

In this paper we reviewed the standard connection between relic abundance and annihilation cross sections, paying particular attention to the uncertainties in the connection. We reviewed how this connection generally results in a lower limit to the rate of DM particles interacting in a detector. We listed the assumptions that go into such a limit and detailed the loopholes and model dependence. While a rate larger than one event per kilogram per day is generally to be expected, we showed that rates between 0 and 1000 events per kilogram per day are possible. Accelerator experiments such as ASP, which attempt to directly produce DM particles also can be used to give general information on galactic DM detection experiments. DM particles under a few GeV are likely to be inconsistent with such experiments although again loopholes in the argument exist.

In conclusion, the connection between the annihilation, elastic and production cross sections can be used in fairly model independent ways to constrain otherwise arbitrary quantities such as the DM mass and interaction rate; however, uncertainties exist and the model dependence can be greater than is commonly recognized.

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APPENDIX A: Cross sections for fermionic dark matter particles

The effective Lagrangian describing the interaction of fermionic Dark Matter (DM) particles with quarks or leptons can in many cases be written

$$L_{eff} = C\bar{\delta}\gamma_\mu(V_\delta + A_\delta\gamma_5)\delta\bar{\psi}\gamma^\mu(V_f + A_f\gamma_5)\psi, \qquad (A1)$$

where C is a constant involving coupling strengths and propagator masses, δ is the DM field, ψ is the quark or lepton field, V_δ and A_δ are the DM vector and axial vector couplings and V_f, A_f are the same for the quark or lepton.

Using eq. A1, the annihilation cross section of two Dark Matter particles of mass m_δ into a pair of fermions of mass m_f, ($\delta\bar{\delta} \to \psi\bar{\psi}$), is found to be

$$\sigma_{ann} = \frac{C'^2 p_f}{4\pi E^2 p_\delta}\Big\{ (V_f^2 + A_f^2)(V_\delta^2 + A_\delta^2)(E^4 + p_f^2 p_\delta^2/3)$$
$$+ m_\delta^2 m_f^2 (V_f^2 - A_f^2)(V_\delta^2 - A_\delta^2) + \frac{m_f^2}{2}(E^2 + p_\delta^2)(V_f^2 - A_f^2)(V_\delta^2 + A_\delta^2)$$
$$+ \frac{m_\delta^2}{2}(E^2 + p_f^2)(V_f^2 + A_f^2)(V_\delta^2 - A_\delta^2)\Big\}, \qquad (A2)$$

where p_δ and p_f are the 3-momenta of the DM and fermion respectively, $E^2 = m_\delta^2 + p_\delta^2 = m_f^2 + p_f^2$, and v is the relative velocity of the incoming particles. In the limit $v \ll 1$ ($c = \hbar = 1$) this can be expanded in powers of v^2

$$\sigma_{ann} v \approx \sum_f \frac{C'^2 m_\delta^2 \sqrt{1-z^2}}{2\pi}\Big\{ 2V_\delta^2(V_f^2 + A_f^2) + z^2(V_\delta^2 V_f^2 + A_\delta^2 A_f^2 - 2V_\delta^2 A_f^2)$$
$$+ \frac{v^2}{3}(V_\delta^2 + A_\delta^2)(V_f^2 + A_f^2) + \frac{v^2 z^2}{12}(2V_\delta^2 A_f^2 - V_\delta^2 V_f^2 + 2A_\delta^2 V_f^2 - 7A_\delta^2 A_f^2)$$
$$+ \frac{v^2 x^2}{2} V_\delta^2 (V_f^2 + A_f^2) + \frac{v^2 z^2 x^2}{4}(V_\delta^2 V_f^2 - 2V_\delta^2 A_f^2 + A_\delta^2 A_f^2)\Big\}, \qquad (A3)$$

where $z^2 = m_f^2/m_\delta^2$, $x^2 = \frac{1}{2}z^2/(1-z^2)$, and the sum is over all fermions f with mass $m_f < m_\delta$. We expand in powers of v^2 since analytic approximations for the relic abundance of δ depend on having a cross section of the form $a + bv^2 + cv^4 + \cdots$. Note, however, that the factor $z^2 v^2/(1-z^2)$ blows up at $z = 1$ and so the expansion is not valid near $m_\delta = m_f$. We will use the expansion anyway, but avoid calculating relic abundances at or just above mass thresholds. The v^2 above is the relative velocity and should be replaced with $6T/m_\delta$ in the thermal average.

Some care must be taken in applying eq. A3 in particular cases. For example, if δ is a Majorana fermion, the self-conjugacy of δ implies $\bar{\delta}\gamma_\mu\delta = 0$ and also an extra factor of two in going from the effective Lagrangian to the Feynman rule. This means $V_\delta = 0$ and an extra factor of four in the cross section. In addition, when the underlying interaction involves the exchange of a massive particle in the s channel, a pole factor should be included. The common example is the Z^0 pole factor

$$P_Z = m_Z^4 / \left[(4m_\delta^2 - m_Z^2)^2 + \Gamma_Z^2 m_Z^2\right], \tag{A4}$$

where m_Z and Γ_Z are the mass and width of the Z^0 boson. Finally there is a color factor $c_f = 3$ when f is a quark. These special case factors in eq. A3 are soaked up by the new constant C'.

Using eq. A1 the elastic scattering cross section of DM particles off fermions can be computed. The matrix element is the same as for annihilation with the replacement $p^\mu_{\delta\, in} \leftrightarrow -p^\mu_{f\, out}$. Here, keeping only the first term in the non-relativistic expansion (which is valid since we are interested in δ's which move with the galactic dispersion velocity), we find

$$\sigma_{el} = \frac{m_\delta^2 m_f^2 C''^2}{\pi(m_\delta + m_f)^2}(V_\delta^2 V_f^2 + 3A_\delta^2 A_f^2). \tag{A5}$$

Here also, the same remarks concerning Majorana particles apply ($V_\delta = 0$ and $C''^2 = 4C^2$), but the color factor is unity even for quarks, and there is no pole factor.

Note that there are no terms involving $A_\delta V_f$ or $A_f V_\delta$ in the elastic scattering cross section, while there are such terms in the annihilation cross section. This means that it is possible for the scattering cross section to be zero while still having substantial annihilation cross section. Also note that in general the annihilation of DM is into quarks and leptons so eq. A3 is applicable, but in elastic scattering, the DM typically scatters off nuclei rather than quarks or leptons and so coherence and other nuclear physics effects must be taken into account. (See the body of the text for details on this point.)

Finally, using the same effective Lagrangian to find $\sigma(e^+e^- \rightarrow \delta\bar{\delta})$ one can find the cross section for $e^+e^- \rightarrow \delta\bar{\delta}\gamma$, in the soft photon limit[27] which is applicable for ASP like experiments.

$$\frac{d\sigma(e^+e^- \rightarrow \gamma\delta\bar{\delta}; s)}{dx\,dy} \approx \frac{2\alpha}{\pi}\frac{\left[(1-\frac{1}{2}x)^2 + \frac{1}{4}x^2y^2\right]}{x(1-y^2)}\sigma(e^+e^- \rightarrow \delta\bar{\delta}; \hat{s}) \tag{A6}$$

where s is the Mandelstam variable, $x = 2E_\gamma/\sqrt{s}$ is the dimensionless photon energy, $y = \cos\theta$ is the angle between the beam and the photon, $\hat{s} = s(1-x)$, and

$$\sigma(e^+e^- \to \delta\bar{\delta}; s) \approx \frac{C'''^2}{4\pi} s\sqrt{1 - 4m_x^2/s}\left\{ \frac{1}{3}(V_\delta^2 + A_\delta^2)(V_f^2 + A_f^2)\left(1 - \frac{m_\delta^2}{s}\right) + (V_\delta^2 - A_\delta^2)(V_f^2 + A_f^2)\frac{m_\delta^2}{s} \right\}, \tag{A7}$$

where C'''^2 is the same as C'^2 above with an extra factor of 1/2 if there are identical particles in the final state.

Eqs. A3, A5, and A7 will not be valid if the interaction cannot be written in the form of eq. A1. This can happen when there are several channels which cannot be combined, or when the Fierz transformation necessary to bring t-channel annihilation into the form of eq. A1 gives tensor or scalar pieces. An example of the latter case is photino DM when the left and right chiral squarks mix.

As examples, we apply the cross section formulas to massive Dirac neutrinos, massive Majorana neutrinos, idealized photinos, and idealized higgsinos (see ref. 22 for details of the supersymmetry model).

For massive **Dirac neutrinos** $C = G_F/\sqrt{2}$, $V_\delta = 1$, $A_\delta = -1$, $V_f = g_V = T_{3f}^L - 2Q_f\sin^2\theta_w$, $A_f = g_A = -T_{3f}^L$, $C'^2 = P_Z c_f C^2$, and $C''^2 = C^2$, where T_{3f}^L and Q_f are the third component of weak isospin and the charge of the quark or lepton (*e.g.* for the up quark, $g_V = \frac{1}{2} - \frac{4}{3}\sin^2\theta_w$ and $g_A = -\frac{1}{2}$), G_F is the Fermi constant, and c_f is the color factor. This gives

$$\langle\sigma v\rangle_{ann} = \sum_f \frac{c_f P_Z m_\delta^2 G_F^2 \sqrt{1-z^2}}{2\pi}\left\{(g_V^2 + g_A^2) + \frac{z^2}{2}(g_V^2 - g_A^2) + \frac{v^2}{3}(g_V^2 + g_A^2) + \frac{v^2 z^2}{24}(g_V^2 - 5g_A^2) + \frac{v^2 x^2}{4}(g_V^2 + g_A^2) + \frac{v^2 z^2 x^2}{8}(g_V^2 - g_A^2)\right\}. \tag{A8}$$

This disagrees with Kane and Kani[28] but agrees with the Kolb and Olive erratum.[29] Apart from coherence factors, the Dirac neutrino elastic scattering cross section is

$$\sigma_{el} = \frac{G_F^2 m_\delta^2 n_f^2 (3g_A^2 + g_V^2)}{2\pi(m_\delta + m_f)^2}, \qquad (A9)$$

which also disagrees with Kane and Kani.

Massive Majorana neutrinos are the same as Dirac neutrinos with $V_\delta = 0$, $C'^2 = 4P_Z c_f C^2$, and $C''^2 = 4C^2$ giving

$$\langle \sigma v \rangle_{ann} = \sum_f \frac{c_f P_Z \sqrt{1-z^2} m_\delta^2 G_F^2}{\pi} \Big\{ z^2 g_A^2 + \frac{v^2}{3}(g_V^2 + g_A^2) + \frac{v^2 z^2}{12} + (2g_V^2 - 7g_A^2) + \frac{v^2 z^2 x^2}{4} g_A^2 \Big\}. \qquad (A10)$$

Eq. A10 differs from the Kolb and Olive erratum and is not given by Kane and Kani. Apart from coherence factors, the elastic scattering cross section for Majorana neutrinos is

$$\sigma_{el} = \frac{6G_F^2 m_\delta^2 m_f^2 g_A^2}{\pi(m_\delta + m_f)^2}. \qquad (A11)$$

For a pure photino[22] we have no vector coupling so $V_f = V_\delta = 0$, $A_f = A_\delta = 1$, and $C = 2\pi\alpha Q_f^2 / M_{sf}^2$ where Q_f is the charge of the fermion and M_{sf} is the mass of the corresponding scalar fermion. We find

$$\langle \sigma v \rangle_{ann} = \sum_f \frac{8\pi\alpha^2 c_f Q_f^4 \sqrt{1-z^2} m_\delta^2}{M_{sf}^4} \left\{ z^2 + \frac{v^2}{3}\left(1 - \frac{z^2}{8}(14 - 8x^2)\right)\right\} \qquad (A12)$$

and apart from spin coherence factors

$$\sigma_{el} = \frac{48\pi\alpha^2 Q_f^4 m_\delta^2 m_f^2}{(m_\delta + m_f)^2 M_{sf}^4}. \qquad (A13)$$

Eq. A12 disagrees with Kane and Kani, but agrees with Srednicki, Olive and Silk.[30] Eq. A13 agrees with Kane and Kani.

Finally, for the idealized **higgsino** discussed in Ellis, *et al.*, we have $V_\delta = 0$, $A_\delta = 1$, $V_f = \cos 2\beta(T^L_{3f} - 2Q_f \sin^2\theta_w)P_Z^{1/2}$, and $A_f = -T^L_{3f}P_Z^{1/2}\cos 2\beta + d_f m_f^2/M^2_{sf}$, where $\tan\beta$ is the ratio of vacuum expectation values of the two Higgs doublets, $d_f = \tan^2\beta$ for the up, charm and top quarks, and $d_f = \cot^2\beta$ for the leptons and down type quarks. For higgsinos both Z^0 and sfermion exchange contribute and we include the Z^0 pole term only on the Z^0 exchange pieces. These factors of P_Z should be left out for elastic scattering. With $C^2 = G_F^2/2$ and the Majorana factor of four we have

$$\langle\sigma v\rangle_{ann} = \sum_f \frac{c_f m_\delta^2 G_F^2\sqrt{1-z^2}}{\pi}\Big\{z^2A_f^2 + \frac{v^2}{3}(V_f^2 + A_f^2) + \frac{v^2z^2}{12}(2V_f^2 - 7A_f^2) + \frac{v^2z^2x^2}{4}A_f^2\Big\} \qquad (A14)$$

and apart from coherence factors

$$\sigma_{el} = \frac{6G_F^2 m_\delta^2 m_f^2 A_f^2}{\pi(m_\delta + m_f)^2} \approx \frac{3G_F^2 m_\delta^2 m_f^2\cos^2 2\beta}{2\pi(m_\delta + m_f)^2}. \qquad (A15)$$

Eq. A15 agrees with Refs. (28) and (30) but Eq. A14 disagrees with Ref. (28).