ABSTRACT

It is suggested that the *Planck* $h = m_k c \lambda_k$ and the *Boltzmann* $k = m_k c v_k$ constants have stochastic foundation. It is further suggested that a body of fluid at equilibrium is composed of a spectrum of molecular clusters (energy levels) the size of which are governed by the *Maxwell-Boltzmann* distribution function. Brownian motions are attributed to equilibrium between suspensions and moelcular clusters. Atomic (molecular) transition between different size atomic- (molecular-) clusters (energy levels) is shown to result in emission/absorption of energy in accordance with *Bohr's* theory of atomic spectra. Physical space is identified as a tachyonic fluid that is *Dirac's* stochastic ether or *de Broglie's* hidden thermostat. Compressibility of physical space , in accordance with *Planck's* compressible ether, is shown to result in the *Lorentz-Fitzgerald* contraction, thus providing a *causal* explanation of relativistic effect in accordance with the perceptions of *Poincaré* and *Lorentz*. The invariant *Schrödinger* equation is derived from the invariant *Bernoulli* equation for incompressible potential flow. Following *Heisenberg* a temporal uncertainty relation is introduced as $\Delta v_\beta \Delta p_\beta \geq k$.

1. INTRODUCTION

What stochastic quantum fields [1-16] and classical hydrodynamic fields [17-26] have in common is their underlying statistical nature. Guided by such unifying features, a scale-invariant model of statistical mechanics was recently introduced [27], and its application to the field of statistical thermodynamics [28, 29] was examined. In the present study, further implications of the model to the foundation of quantum mechanics, the special theory of relativity and relativistic thermodynamics will be investigated.

2. A SCALE-INVARIANT MODEL OF STATISTICAL MECHANICS

Following the classical methods [30-34] the invariant definition of density ρ_β, and velocity of *element* v_β, *atom* u_β, and *system* w_β at the scale β are [28]

$$\rho_\beta = n_\beta m_\beta = m_\beta \int f_\beta du_\beta \quad , \quad v_\beta = \rho_\beta^{-1} m_\beta \int u_\beta f_\beta du_\beta \tag{2.1}$$

$$u_\beta = v_{\beta-1} \quad , \quad w_\beta = v_{\beta+1} \tag{2.2}$$

Similarly, the invariant definition of the peculiar and diffusion velocities are introduced as

$$V'_\beta = u_\beta - v_\beta \quad , \quad V_\beta = v_\beta - w_\beta \tag{2.3}$$

such that

$$V_\beta = V'_{\beta+1} \tag{2.4}$$

3. STOCHASTIC NATURE OF THE PLANCK AND THE BOLTZMANN CONSTANTS

Because at the state of thermodynamic equilibrium $<u_\beta> = 0$, the energy of each particle (oscillator) is expressed as

$$\varepsilon_\beta = m_\beta<u^2{}_\beta> = m_\beta<\lambda_\beta\nu_\beta\lambda_\beta\nu_\beta> = m_\beta<(\lambda_\beta\nu_\beta)^2>^{1/2}<(\lambda_\beta\nu_\beta)^2>^{1/2}$$
$$= m_\beta<u^2{}_\beta>^{1/2}<\lambda_\beta{}^2>^{1/2}<\nu_\beta{}^2>^{1/2} = <p_\beta><\lambda^2{}_\beta>^{1/2}<\nu^2{}_\beta>^{1/2} \qquad (3.1)$$

where $m_\beta<u^2{}_\beta>^{1/2} = <p_\beta>$. The above result could be expressed in either forms

$$\varepsilon_\beta = m_\beta<u^2{}_\beta> = <p_\beta><\lambda^2{}_\beta>^{1/2}<\nu^2{}_\beta>^{1/2} = h_\beta<\nu^2{}_\beta>^{1/2} = h_\beta<\nu_\beta> \qquad (3.2)$$

$$\varepsilon_\beta = m_\beta<u^2{}_\beta> = <p_\beta><\nu^2{}_\beta>^{1/2}<\lambda^2{}_\beta>^{1/2} = k_\beta<\lambda^2{}_\beta>^{1/2} = k_\beta<\lambda_\beta> \qquad (3.3)$$

when the definition of stochastic *Planck* and *Boltzmann* factors are introduced as

$$h_\beta = <p_\beta><\lambda^2{}_\beta>^{1/2} \qquad (3.4)$$

$$k_\beta = <p_\beta><\nu^2{}_\beta>^{1/2} \qquad (3.5)$$

At the important scale of EKD one obtains the universal constants of *Planck* [35, 36] and *Boltzmann* [28] that in view of (3.4)-(3.5) become

$$h = h_k = <p_k><\lambda^2{}_k>^{1/2} = m_k c<\lambda^2{}_k>^{1/2} = 6.626\times10^{-34} \quad \text{J-s} \qquad (3.6)$$

$$k = k_k = <p_k><\nu^2{}_k>^{1/2} = m_k c<\nu^2{}_k>^{1/2} = 1.381\times10^{-23} \quad \text{J/K} \qquad (3.7)$$

Following *de Broglie* hypothesis for the wavelength of matter waves [2]

$$\lambda_\beta = h/p_\beta \qquad (3.8)$$

the frequency of matter waves was introduced as [28]

$$\nu_\beta = k/p_\beta \qquad (3.9)$$

Under thermal equilibrium between matter and radiation (3.8)-(3.9) are expressed as

$$h_\beta = h_k = h \quad \text{and} \quad k_\beta = k_k = k \qquad (3.10)$$

The definitions (3.6) and (3.7) result in the gravitational mass of photon [28]

$$m_k = (hk/c^3)^{1/2} = 1.84278\times10^{-41} \text{ g} \qquad (3.11)$$

that is much larger than the reported value of 4×10^{-51} kg [37]. The finite gravitational mass of photons was anticipated by *Newton* [38] and is in accordance with the *Einstein-de Broglie* theory of light [39-41]. The *Avogardo-Loschmidt* number is predicted as [28]

$$N^\circ = 1/(m_k c^2) = 6.0376\times10^{23} \text{ [molecules/g-mole]} \qquad (3.12)$$

leading to the universal gas constant $R^\circ = N^\circ k = 8.3379$ [J/ (g-mole . K)].

The rest energy $\varepsilon_{o\beta}$ of a stationary particle at scale β may be expressed as the sum of the energies of the atoms composing it

$$\varepsilon_{o\beta} = \Sigma\varepsilon_{\beta-1} = \Sigma kT_{\beta-1} = \Sigma k<\lambda_{\beta-1}> = N_{\beta-1}k<\lambda_{\beta-1}> = k<\lambda_\beta> = kT_\beta \qquad (3.13)$$

such that

$$\varepsilon_{o\beta} = \Sigma\varepsilon_{\beta-1} = N_{\beta-1}\varepsilon_{\beta-1} = N_{\beta-1}\Sigma\varepsilon_{\beta-2} = N_{\beta-1}N_{\beta-2}\varepsilon_{\beta-2} =$$
$$= \ldots = N_{\beta-1}N_{\beta-2}\ldots N_k\varepsilon_k = N_{\beta-1}N_{\beta-2}\ldots N_k m_k c^2 = m_{o\beta}c^2 \qquad (3.14)$$

leading to the definition of gravitational mass

$$m_{o\beta} = N_{\beta-1}N_{\beta-2}\ldots N_k m_k \qquad (3.15)$$

This is in harmony with the perceptions of *Weyl* [42] who associated mass with pure numbers.

Because of the definition of atomic mass unit in (3.12), every particle can be considered as a virtual oscillator with energy uncertainty

$$\Delta\varepsilon_\beta = \Delta\lambda_\beta\,\Delta p_\beta\,\Delta\nu_\beta = \Sigma_{\beta-1}\Sigma_{\beta-2}\ldots\Sigma_k\,\varepsilon_k = N_{k\beta}\,h\,\Delta\nu_\beta \qquad (3.16)$$

Since number of photons in the particle must always satisfy $N_{k\beta} \geq 1$, one arrives at the *Heisenberg* [43] *spatial uncertainty principle*

$$\Delta\lambda_\beta\,\Delta p_\beta \geq h \qquad (3.17)$$

setting a limit on spatial resolution of position measurements. But (3.16) could also be expressed in terms of wavelength and *Boltzmann* constant by (3.3) as

$$\Delta \varepsilon_\beta = \Delta \nu_\beta \, \Delta p_\beta \, \Delta \lambda_\beta = \Sigma_{\beta-1} \Sigma_{\beta-1} \ldots \Sigma_k \, \varepsilon_k = N_{k\beta} \, k \, \Delta \lambda_\beta \tag{3.18}$$

that with criteris satisfy $N_{k\beta} \geq 1$ leads to the *temporal uncertainty principle* [29]

$$\Delta \nu_\beta \, \Delta p_\beta \geq k \tag{3.19}$$

thus setting a limitation on temporal resolution of time measurements.

4. THE NATURE OF BROWNIAN MOTIONS AND RESOLUTION OF MAXWELL'S DEMON PARADOX AND THE QUANTIZATION PROBLEM

The evidence for the existence of the statistical field of equilibrium cluster-dynamics ECD is the phenomena of Brownian motions [23, 44-50]. Modern theory of Brownian motion starts with the *Langevin* equation [23].

$$\frac{du_p}{dt} = -\beta u_p + A(t) \tag{4.1}$$

where $\mathbf{u}_p$ is the particle velocity. The drastic nature of the assumptions inherent in the division of forces in Eq.(4.1) was emphasized by *Chandrasekhar* [23].

To account for the stationary nature of Brownian motions, fluid fluctuations at scales much larger than molecular scales are needed as noted by *Gouy* [50]. Observations have shown that as the size of the particles decrease their movement become faster [47]. The experimental measurement of *Maxwell-Boltzmann* velocity distribution [51] only reveal the fraction of total number of atoms with velocity v by determining the intensity of ionic flux induced by collision of neutral atoms. Let us consider a heated oven containing a total of N_{ac} atoms under random motions, and let N_{amj} be the number of atoms in atomic-cluster j (i.e. "molecule" j) that have the particular velocity u_{aj}. If one now adjusts the phase and the rotation velocity of the two rotating disks in the experiment [51] for the velocity u_{aj}, one will expect the relative number of atoms N_{amj}/N_{ac} to vary like the measured relative intensity of ionic flux (I_j/I_o). Therefore, one can argue that the measured number of atoms at any given atomic-cluster (molecular) velocity u_{amj} denotes the fraction of atomic-clusters of *particular size* containing N_{amj} atoms with the velocity $u_{mj} = \langle u_{aj} \rangle = u_{aj}$. Thus, *Maxwell-Boltzmann* statistical field gives spectral distribution of atomic-cluster sizes, and hence velocities, within the system as schematically shown in Fig.1 for the temperature T' = 300 K and arbitrary vertical scale.

At molecular-dynamic scale $\beta = m$, the system is an eddy that is composed of a spectrum of molecular-clusters from the smallest cluster that is a single molecule moving at the maximum molecular velocity of $w_s = v_a = u_m \approx 1200$ m/s to the largest cluster that is the eddy itself moving at the minimum molecular velocity $w_m = v_c = u_e \approx 0.11$ m/s associated with the harmonic oscillations of the system (Fig.1). Simultaneously, but at the much smaller scale of EAD $\beta = a$, one has a spectrum of sub-particle clusters (i.e. atoms) varying in size from a single sub-particle moving at the maximum atomic velocity of about $v_s = u_a \approx 3200$ m/s (Fig.1) to the size of the system that is a molecular cluster moving at the minimum atomic velocity of about $w_a = v_m = u_c \approx 360$ m/s (Fig.1). In the schematic diagram in Fig.1, while T is being held constant the mass of the particles is varied from m_s, to m_c. Also, the velocity scale between 0.1 and zero is expanded for clearer visualization.

The energy of all atomic clusters ("molecules") are considered to be identical under thermodynamic equilibrium at a given temperature T such that

$$\varepsilon_{mj} = kT_{mj} = m_{mj} \langle u^2{}_{mj+} \rangle = \varepsilon_{mi} = kT_{mi} = m_{mi} \langle u^2{}_{mi+} \rangle = \varepsilon_a = kT_a = kT \tag{4.2}$$

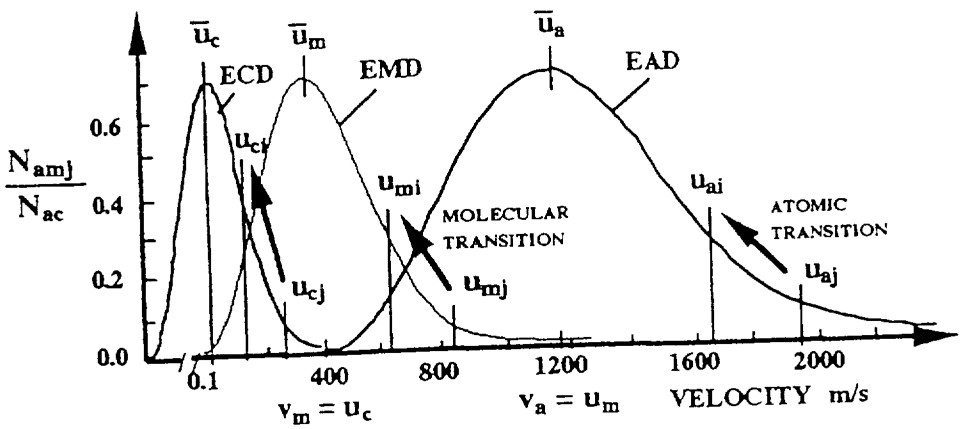

Fig.1 Maxwell-Boltzmann velocity distribution viewed as stationary spectra of molecular, atomic, and sub-particle cluster sizes at 300 K.

Therefore, the classical paradox of *Maxwell's demon* will no longer be encountered. This is because at equilibrium particles of all size will have the same energy kT, making selection of more energetic particles by the demon impossible. The modified definition of temperature T and the classical one T ' are related by T' = 2T [28]. The smallest molecular cluster corresponds to a single molecule $3kT'_m = 6kT_m = m_m <(u_m)^2> = 2m_m <(u_{m+})^2>$. In atmospheric air with $T'_m = 2T_m = 300$ K and $m_m = 28.9 \times 1.656 \times 10^{-27}$ kg by (3.14), the mean molecular speed will be about $\bar{u}_{m+} \approx 360$ m/s that is in close agreement with the measured speed of sound in standard atmosphere as initially suspected by *Newton*.

Considering atomic cluster (i.e. "molecule") with smaller mass $m_{mj} = N_{amj}\, m_a < m_{mj} = N_{amj}\, m_a$ will have larger harmonic velocity $u_{mj} > u_{mi}$ since $T_{mj} = T_{mi}$ as schematically shown in Fig.1. The factors N_{amj} and N_{ami} refer to the number of atoms in the cluster (j) and (i), respectively. If clusters are modeled as rigid bodies, the energy of the small cluster (j) and the large cluster (i) may be expressed as the sum of the energies of the atoms composing them

$$\varepsilon_{mj} = kT_{mj} = m_{mj} <u^2_{mj}> = \Sigma m_a <u^2_{aj}> = N_{amj}\, \varepsilon_{aj} = N_{amj}\, h <\nu_{aj}> \tag{4.3}$$

$$\varepsilon_{mi} = kT_{mi} = m_{mi} <u^2_{mi}> = \Sigma m_a <u^2_{ai}> = N_{ami}\, \varepsilon_{ai} = N_{ami}\, h <\nu_{ai}> \tag{4.4}$$

The condition $N_{amj} < N_{ami}$ along with (4.2) result in

$$N_{amj} <\nu_{aj}> = N_{ami} <\nu_{ai}> \quad \text{and} \quad \varepsilon_{aj} > \varepsilon_{ai} \tag{4.5}$$

Therefore, transfer of an atom from (j) to (i) cluster, that is equivalent to the *transition* from the *high-energy-level* (j) to the *low-energy-level* (i) shown in Fig.1, by (3.2) releases the energy in accordance with *Bohr's* theory of atomic spectra [52]

$$\Delta \varepsilon_{aji} = \varepsilon_{aj} - \varepsilon_{ai} = h [<\nu_{aj}> - <\nu_{ai}>] \tag{4.6}$$

Therefore, the reason for quantum nature of the energy of *atomic spectra* is that only transitions between clusters (energy levels) are allowed that themselves must satisfy a criteria of *stationarity* imposed by the *Maxwell-Boltzmann* distribution (Fig.1).

5. IDENTIFICATION OF SPACE AS A COMPRESSIBLE TACHYONIC FLUID AND ITS IMPACT ON THE SPECIAL THEORY OF RELATIVITY

Photons are considered to be composed of a cluster of much smaller particles [28] called *tachyons* [53]. The *physical space* is identified as a *tachyonic fluid* in accordance with *Dirac's* stochastic ether [54] and *de Broglie's* "hidden thermostat" [3]. The central importance of a medium called "ether" to the theory of electrons was emphasized by *Lorentz* [55, 56]

> **" I cannot but regard the ether, which is the seat of an electromagnetic field with its energy and its vibrations, as endowed with certain degree of substantiality, however different it may be from all ordinary matter,"**

Since the velocity of light is the mean thermal speed of tachyons, $u_k = c = v_t$, at least some of the tachyons must be *superluminal*. Tachyonic fluid is considered to be *compressible*, in accordance with *Planck's* compressible ether [56]. Compressibility of space is evidenced by the fact that the velocity of light is finite $c < \infty$. To further reveal the analogy between compressible tachyonic fluid and compressible ideal gas, we consider the change of density of an ideal gas brought isentropically to the state of rest given by

$$\rho = \rho_o\left[1 + \frac{\gamma - 1}{2}\left(\frac{v}{a}\right)^2\right]^{\frac{1}{\gamma - 1}} = \rho_o\left[1 + \frac{\gamma - 1}{2}\, Ma^2\right]^{\frac{1}{\gamma - 1}} \tag{5.1}$$

involving the *Mach* number $Ma = v/a$, the velocity of sound a, and the specific heat ratio $\gamma = c_p/c_v$. For the tachyonic fluid, one may express the above relation in terms of the *Michelson* number $Mi = v/c$

$$\rho = \rho_o\left[1 + \frac{\gamma - 1}{2}\left(\frac{v}{c}\right)^2\right]^{\frac{1}{\gamma - 1}} = \rho_o\left[1 + \frac{\gamma - 1}{2}\, Mi^2\right]^{\frac{1}{\gamma - 1}} \tag{5.2}$$

Using $\gamma = 4/3$ for photon gas, one obtains

$$\rho = \rho_o\left[1 + \frac{1}{6}Mi^2\right]^3 \approx \rho_o\left[1 + \frac{1}{2}Mi^2\right] \approx \frac{\rho_o}{\sqrt{1 - Mi^2}} = \frac{\rho_o}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} \tag{5.3}$$

If one now notes that mass is relativistically invariant $M = M_o$, the densities of moving and stationary fluid could be expressed as

$$\rho = M/V \quad , \quad V = L_x L_y L_z = L_x A \quad , \quad A = L_y L_z \tag{5.4}$$

$$\rho_o = M/V_o \quad , \quad V_o = L_{ox} L_y L_z = L_{ox} A \quad , \quad A = L_y L_z \tag{5.5}$$

Since transverse coordinates do not change [57] for one-dimensional flow A = constant, by (5.3)-(5.5) one arrives at the classical expression of *Lorentz-Fitzgerlad* contraction [56, 58]

$$L_x = L_{ox}\sqrt{1 - \left(\frac{v}{c}\right)^2} \tag{5.6}$$

Thus, supersonic $Ma > 1$ (superchromatic $Mi > 1$) flow of air (tachyonic fluid) leads to the formation of *Mach* (*Minkowski*) cone that separates the zone of sound (light) from the zone of silence (darkness). Compressibility of physical space can therefore account for the *Lorentz-Fitzgerald* contraction [59], hence providing a *causal explanation* of relativistic effects [60] in accordance with the perceptions of *Poincaré* and *Lorentz* [58, 61, 62]. The state of equilibrium of tachyonic fluid that is stochastically homogeneous and isotropic provides for the *absolute inertial frame of reference* that is stochastically stationary with respect to the stochastic isotropic background cosmic radiation [39, 63].

Because of the definition of *Boltzmann* constant (3.7), thermodynamic temperature

(3.12) is identified as a length scale $T_\beta = <\lambda^2{}_\beta>^{1/2}$, and by (5.6) one obtains

$$<\lambda> = <\lambda_o> \sqrt{1 - \left(\frac{v}{c}\right)^2} \qquad \text{and} \qquad T = T_o \sqrt{1 - \left(\frac{v}{c}\right)^2} \tag{5.7}$$

in agreement with classical results [57]. The constant light velocity and (5.7) lead to

$$<v> = <v_o>/\sqrt{1 - \left(\frac{v}{c}\right)^2} \tag{5.8}$$

in accordance with the classical results [57].
Also, since entropy defined as $S = 3Nk$ [28] is relativistically invariant $S = S_o$ [57], with reversible heat defined as $Q = TS$ [28] and (5.7) one obtains

$$Q = Q_o \sqrt{1 - \left(\frac{v}{c}\right)^2} \tag{5.9}$$

that is in accordance with the classical result [3, 57].

For particle moving at velocity v the total energy will be

$$\varepsilon = m_o c^2 + m_o v^2/2 \tag{5.10}$$

with m_o defined in (3.15). One may then express the preceding relation as

$$\varepsilon = m_o c^2 \left[1 + (v^2/c^2)/2\right] \approx m_o c^2/\sqrt{1 - \left(\frac{v}{c}\right)^2} = \varepsilon_o/\sqrt{1 - \left(\frac{v}{c}\right)^2} \tag{5.11}$$

in accordance with the classical result [57]. With the relativistic mass m_r [58]

$$m_r \approx m_o/\sqrt{1 - \left(\frac{v}{c}\right)^2} \tag{5.12}$$

the energy relation (5.11) could also be expressed as

$$\varepsilon = m_r c^2 \tag{5.13}$$

It appears that according to (5.11), as the particle velocity approaches the velocity of light $v \rightarrow c$, the particle relativistic mass becomes infinite $m_r \rightarrow \infty$. However, this problem is caused by the approximation introduced between the second and the third terms of (5.13). In fact, from the second term of (5.11) one obtains in the limit $v \rightarrow c$

$$\varepsilon = m_o c^2 \left[1 + (v^2/c^2)/2\right] = (3/2)\ m_o c^2 = (3/2)\, kT \tag{5.14}$$

and hence $m_r = (3/2) m_o$ that is finite. The metamorphosis of matter into light and vice-versa was first noted by *Newton* [29, 38] and is a common occurrence in combustion such as burning of a match stick.

The finite gravitational mass of photon given in (3.11) is also in accordance with the perceptions of *Poincaré* who first introduced [65, 66]

$$E = mc^2 \tag{5.15}$$

corresponding to (5.13) where $m = m_r$ is the relativistic mass. The expression introduced later by *Einstein* [67, 68] for the rest energy of particle

$$E_o = mc^2 \tag{5.16}$$

corresponds to (3.15), and was conceived under the assumption of massless photons.

6. DERIVATION OF THE SCALE-INVARIANT SCHRÖDINGER EQUATION FROM THE INVARIANT BERNOULLI EQUATION

Following the classical methods [30-34], the scale-invariant forms of mass and linear momentum conservation equations are given as [27]

$$\frac{\partial \rho_\beta}{\partial t} + \nabla \cdot (\rho_\beta v_\beta) = 0 \tag{6.1}$$

$$\frac{\partial (\rho_\beta v_\beta)}{\partial t} + \nabla \cdot (\rho_\beta v_\beta v_\beta) = 0 \tag{6.2}$$

For an irrotational $v_\beta = \nabla\Phi_\beta$ and incompressible fluid with the velocity potential Φ_β, (6.1)-(6.2) lead to the invariant *Bernoulli* equation

$$\partial(\rho_\beta\Phi_\beta)/\partial t + (\nabla\rho_\beta\Phi_\beta)^2/(2\rho_\beta) = \text{Constant} = 0 \quad (6.3)$$

Comparison of (6.3) with the *Hamilton-Jacobi* equation [2, 69-71]

$$\partial S/\partial t + (\nabla S)^2/2m + V = 0 \quad (6.4)$$

leads to the invariant definition of the modified action [72]

$$S_\beta(\mathbf{x}, t) \equiv \rho_\beta\Phi_\beta \quad (6.5)$$

The relation (2.3) between the velocities $v_\beta = u_\beta - V'_\beta$ suggests that

$$\Phi_\beta(\mathbf{x}, t) = \Phi_{o\beta}(\mathbf{x}) - \varepsilon\Phi'_\beta(\mathbf{x}, t) \qquad \varepsilon << 1 \quad (6.6)$$

since $\nabla\times v_\beta = 0 = \nabla\times u_\beta - \nabla\times V'_\beta = 0$ leads to $u_\beta = \nabla\Phi_{o\beta}$ and $V'_\beta(\mathbf{x}, t) = \varepsilon\nabla\Phi'_\beta$. Therefore, (6.5) and (6.6) give

$$S_\beta(\mathbf{x}, t) = S_{o\beta}(\mathbf{x}) - \varepsilon S'_\beta(\mathbf{x}, t) = S_{o\beta}(\mathbf{x}) - \varepsilon\Psi_\beta(\mathbf{x}, t) \quad (6.7)$$

with the *quantum mechanics wave function* $\Psi_\beta(\mathbf{x}, t)$ defined as [72]

$$\Psi_\beta(\mathbf{x}, t) \equiv S'_\beta(\mathbf{x}, t) = \rho_\beta\Phi'_\beta(\mathbf{x}, t) \quad (6.8)$$

Substituting from (6.8) into (6.3) and separating terms of equal powers of ε leads to

$$\frac{\partial S_{o\beta}}{\partial t} + \frac{(\nabla S_{o\beta})^2}{2\rho_\beta} + \widetilde{V}_\beta = 0 \quad (6.9)$$

$$\frac{\partial\Psi_\beta}{\partial t} + \frac{\nabla S_{o\beta}\nabla\Psi_\beta}{\rho_\beta} = 0 \quad (6.10)$$

where the *volumetric potential energy density* is defined as [28]

$$\widetilde{V}_\beta \equiv \varepsilon^2(\nabla\Psi_\beta)^2/(2\rho_\beta) = \rho_\beta V'^2_\beta/2 = n_\beta\overline{V}_\beta \quad (6.11)$$

To analyze (6.9), one introduces the coordinate $z = x - u_\beta t$ such that

$$\nabla_x S_{o\beta} = \nabla_z S_{o\beta} \qquad \text{and} \qquad \frac{\partial S_{o\beta}}{\partial t} = -u_\beta\nabla_z S_{o\beta} \quad (6.12)$$

Substitutions from (6.7), (6.11) and (6.12) into (6.9) gives

$$\widetilde{E}_\beta = \widetilde{T}_\beta + \widetilde{V}_\beta \quad (6.13)$$

where

$$\widetilde{E}_\beta \equiv \rho_\beta u^2_\beta = n_\beta\overline{E}_\beta \quad , \quad \widetilde{T}_\beta \equiv \rho_\beta u^2_\beta/2 = n_\beta\overline{T}_\beta \quad , \quad \widetilde{V}_\beta = \rho_\beta u^2_\beta/2 = n_\beta\overline{V}_\beta \quad (6.14)$$

Next, equation (6.10) is treated by taking its first time derivative and substituting for $\partial\Psi_\beta/\partial t$ from (6.10) itself to obtain the wave equation

$$\frac{\partial^2\Psi_\beta}{\partial t^2} = u^2_\beta\,\nabla^2\Psi_\beta \quad (6.15)$$

With the product solution $\Psi_\beta(\mathbf{x}, t) = \psi_\beta(\mathbf{x})\,\Lambda_\beta(t)$ in (6.15) one obtains

$$\psi_\beta''/\psi_\beta = \Lambda''_\beta/(\Lambda_\beta u^2_\beta) = -\sigma_\beta^2 \quad (6.16)$$

when σ_β is the separation constant. The solution of temporal part of (6.16) is

$$\Lambda_\beta = \exp(-i\sigma_\beta u_\beta t) = \exp(-i\omega_\beta t) \quad (6.17)$$

suggesting that $\sigma_\beta u_\beta = \omega_\beta = 2\pi\nu_\beta$ is a circular frequency ω_β.

Following *Planck* [35], one introduces by (3.2) the invariant expressions

$$\overline{E}_\beta = m_\beta u^2_\beta = m_\beta\lambda_\beta u_\beta\nu_\beta = h_\beta\nu_\beta \qquad \text{and} \qquad P_\beta = \widetilde{E}_\beta = n_\beta h_\beta\nu_\beta \quad (6.18)$$

when the harmonic atomic velocity is expressed $u_\beta = \lambda_\beta\nu_\beta$.

The fact that amongst (c, e, h), the *Planck* constant h may be found to be related to other fundamental constants of physics was anticipated by *Dirac* [73]. Using the result in (6.18), one obtains (6.19)

$$\sigma_\beta u_\beta = 2\pi \overline{E}_\beta / h_\beta$$

such that (6.17) becomes (6.20)

$$\Lambda_\beta = \exp\left(-\,i2\pi\, t\, \overline{E}_\beta / h_\beta\right)$$

By substitution from (6.14), (6.19), and (3.10) into (6.16), one obtains the invariant time-independent *Schrödinger* equation [74]

$$\nabla^2 \psi_\beta + \frac{8\pi^2 m_\beta}{h^2}\left(\overline{E}_\beta - \overline{V}_\beta\right)\psi_\beta = 0 \qquad (6.21)$$

and through multiplication by (6.20), the invariant time-dependent *Schrödinger* equation

$$\frac{ih}{2\pi}\frac{\partial \Psi_\beta}{\partial t} + \frac{h^2}{8\pi^2 m_\beta}\nabla^2 \Psi_\beta - \overline{V}_\beta \Psi_\beta = 0 \qquad (6.22)$$

that governs the dynamics of particles from cosmic to tachyonic scales [28]. Since (6.13)-(6.14) lead to $\overline{E}_\beta - \overline{V}_\beta = \overline{T}_\beta$, the *Schrödinger* equation (6.21) gives the *stationary states* of particles that are trapped within a *wave-packet* under the *potential* defined in (6.11) acting as *Poincaré* stress.

7. THE QUANTUM AND PHYSICAL NATURE OF SPACE AND TIME AND THEIR IMPACT ON THE DOUBLE-SLIT AND EPR PARADOXES OF QUANTUM MECHANICS

The physical space, i.e. *vacuum* [39, 75], is identified as a tachyonic fluid and the mean-free-path and the mean-free-frequency of photons in EKD are [28]

$$<\lambda_k> = 1/R^o = 0.119935\,\text{m} \quad , \quad <\nu_k> = 2.49969\times10^9\ \text{Hz} \qquad (7.1)$$

and in view of the definition of temperature in (3.12)

$$T_k = <\lambda_k> = 0.119935\,\text{K} \qquad (7.2)$$

to be compared with cosmic background radiation temperature of 2.73 K. As is common practice in modern physics, *forces* will be associated with the exchange of *atomic particles* for each statistical field from cosmic to tachyonic scale [28]. Therefore, galaxies are exchange particles for cosmic-forces, . . . , clusters for eddy-dynamic forces, . . . , and photons for electro-dynamic forces. Tachyon particles are responsible for high energy tachyonic-forces as well as the gravitational forces in such a stochastic model of quantum gravity [76, 77]. Hence, rather than the four fundamental forces namely gravitational, electro-magnetic, weak, and strong forces, the model suggests an infinite cascade of fundamental forces for $\beta = \ldots g, p, h, f, e, c, m, a, s, k, t, \ldots$ statistical fields. Because of the *stochastic* motion of its atoms (tachyons), *discrete* nature of its fabric, variability of its *measure* (density hence curvature), and *vectorial* nature of its atomic velocity, the *physical space* being presented herein may be described by a *generalized geometry* that may be called *Stochastic-Quantum-Riemannian-Hilbert-Space*. The model [28] also provides a framework for closure of the gap between statistical and continuum mechanics, as well as the quantum field theory and the quantum theory of gravitation [76, 77].

Photon particles are considered as a large ensemble of tachyons [53], and if one assumes each tachyon to have a mass of 10^{-56} kg, in view of (3.11), each photon will contain 10^{12} tachyons. Therefore, under thermal equilibrium between photons and tachyons $kT_k = m_k<u^2{}_k> = kT_t = m_t<u^2{}_t>$, the thermal speed of a single tachyon will be

$<u^2_t>^{1/2} = 10^6\,c = 2.998\times10^{14}$ m/s. This result is in accordance with the perceptions of *Laplace* who considered that universal gravitational signals are transmitted a million times more rapidly than light [61]. Therefore, *apparent* non-local effects may occur that are *causally-connected* through super-luminal interactions [39, 78, 79]. Superluminal transmissions do not cause any causal difficulties, but could lead to *apparent* causality violation if judgments concerning *simultaneity* are made by either luminal or sub-luminal signals.

The motion of photons in the tachyonic fluid (space) will result in the formation of waves in the latter. Following *Maxwell's* kinetic theory of gases, the diffusivity of photon is expected to be $D_k = \lambda_k c/3 = h/(3m_k)$ that is in exact agreement with the prediction of *de Broglie* [3]. According to the theory described in sec.6 [80], the quantum mechanic *wave function* $\Psi_\beta = \rho_\beta \Phi'_\beta$ is defined as the first perturbation of the action $S_\beta = \rho_\beta \Phi_\beta$, that is the product of density and velocity potential. It is thus clear that such a theory resolves the classical paradox of *double-slit* experiment [81]. This is because photons are "guided" [2] by the physical waves on the fabric of space and the maxima of Ψ_β will coincide with maximum density $\rho_\beta = (\Psi_\beta \Psi^*_\beta)^{1/2}$. Double-slit experiments performed in liquid tanks with macroscopic particles could test the validity of this analogy. Similarly, the EPR paradox will be resolved because superluminal interactions [2, 39, 78, 79], responsible for the *apparent* non-locality (action-at-a-distance) in conventional quantum mechanics, are possible by the action of tachyonic signals.

Similar to the spectra of wavelengths, the scale-invariant model of statistical mechanics [28] also provides a spectra of times [27]. Each statistical field possesses (atomic, elemental, system) times ($t_\beta = l_\beta/u_\beta, \tau_\beta = \lambda_\beta/v_\beta, \Theta_\beta = L_\beta/w_\beta$) where l_β and λ_β are the free paths of atoms and elements, and $L_\beta = \lambda_{\beta+1}$ is the system size. Therefore, there exists an *internal clock* associated with random thermal motions of the atoms $t_\beta = \tau_{\beta-1}$ for each statistical field from cosmic to tachyonic scales [28]. As an example, for human beings as biological systems, the most natural time will be the molecular-dynamic time (t_m) that controls the rates of chemical reactions in the body. For instance, if one cools a piece of meat to extremely low temperatures, one slows down the biological time for reactions in the meat thereby prolonging its state of useful lifetime. One may associate the absolute mathematical time of *Newton* to the equilibrium state of tachyon-dynamics (t_t) that in the absence of any non-homogeneities (light) will be a timeless (eternal) world of darkness, irrespective of its stochastic dynamics. The model also reveals the nature of the "internal clock" within each particle and the reason for *phase-coincidence* between the external versus the internal clocks [3, 39, 82-84]. Since time is identified as a physical attribute of the dynamics of tachyonic atoms and space is identified as this compressible tachyonic fluid itself, it is clear that the causal connections between space and time in relativistic physics become apparent. For example, in the classical problem of *twin paradox* of the special theory of relativity, the different times experienced by the twins could be attributed to the different rates of biological reactions in their body induced by the compressibility of physical space. Since matter is composed of photons that are composed of tachyons, the field theory being described [28] is free of singularities.

8. CONCLUDING REMARKS

A scale-invariant model of statistical mechanics was applied to describe the stochastic definitions of *Planck* and *Boltzmann* constants. Transfer of atoms from a small fast-oscillating cluster to a large slow-oscillating cluster, corresponding to the transition from high to low energy levels, was shown to be in accordance with *Bohr's* theory of atomic spectra. Physical space was identified as a tachyonic fluid that is the stochastic ether of *Dirac*, and hidden-thermostat of *de Broglie*. Compressibility of physical space was shown to result in *Lorentz-Fitzgerald* contraction thus providing a *causal* explanation of relativistic effects in accordance with the perceptions of *Poincaré and Lorentz*. The invariant *Schrödinger* equation was *derived* from the invariant *Bernoulli* equation for incompressible potential flow.

ACKNOWLEDGMENTS

This research is supported by NASA grant No. NAG3-1863.

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