

I. INTRODUCTION

The author has been developing a motor operated by the discharge of a capacitor in an *LCR* (inductance-capacitance-resistance) circuit. Unlike conventional dc motors, this motor utilizes the magnetic force of attraction between a current-carrying coil and a movable ferromagnetic core. The force of attraction between the two components resulting from the capacitor discharge is converted to a rotary force. The unconsumed magnetic energy is recycled as electrical energy by recharging the capacitor.

In the course of developing this motor, it was discovered that the recharge voltage depends on the precise configuration of the system.

The purpose of this paper is to describe the increased voltage phenomenon observed in the above system. A differential equation that expresses the phenomenon, as well as computer simulations, are also discussed.

It is appropriate here to briefly discuss other machines based on a similar magnetic phenomenon. Many attempts have been made to operate machinery that utilizes the nonlinear phenomenon of magnetism, such as ferroresonance^{1,2} and parametric resonance.³ The basic features of these machines is the magnetic saturation effect. The machines primarily make use of the transition from a nonresonant state to a resonant state, i.e., from the high inductance of a nonsaturated state to the low inductance of a saturated state, converting these two modes to either oscillation or amplification.

It should be noted that the present system is completely different from these machines, since there is no magnetic saturation in the coils. Voltage changes found in the system occur during the transition from a low-inductance state to a high-inductance state, and are not subjected to the sudden drop or rise typically associated with ferroresonance and parametric resonance. In other words, other systems operate in a closed magnetic field, whereas the system described here operates in an open magnetic field. Electrically, this system is basically closed, since the only power source used here is a charged capacitor; it has no ac power supply such as that used to operate other magnetic machines.

II. *LCR* CIRCUIT WITH AN INCREASE IN INDUCTANCE

The basis of the system discussed in the present paper is a conventional *LCR* circuit. Figure 1(a) shows a basic *LCR*

circuit containing a capacitor initially charged to a voltage of $+V_0$. When the circuit is closed, the capacitor discharges its energy through the inductor. The voltage and current in this transient state are known to follow a damped oscillation [Fig. 1(b)].

Switch *S* can be replaced by a SCR (silicon controlled rectifier) in order to eliminate switching loss [Fig. 2(a)]. The other advantage of using the SCR is that a negative charge in the capacitor is retained after discharge. The oscillation stops after the first discharge, since the SCR automatically turns off when the half-cycle current recharges capacitor *C* to a recharge voltage of $-V_r$. The voltage and current during this process are shown in Fig. 2(b). The amount of recharge voltage is always smaller than the initial voltage due to the resistance loss in the circuit.

The inductor (coil) in Fig. 2(a) is now replaced by two separate coils that face each other, with a movable ferromagnetic core inserted between the coils (Fig. 3). When the two coils (electromagnets) L_1 and L_2 are connected in series, they generate magnetic fields that attract the ferromagnetic core toward the coils.

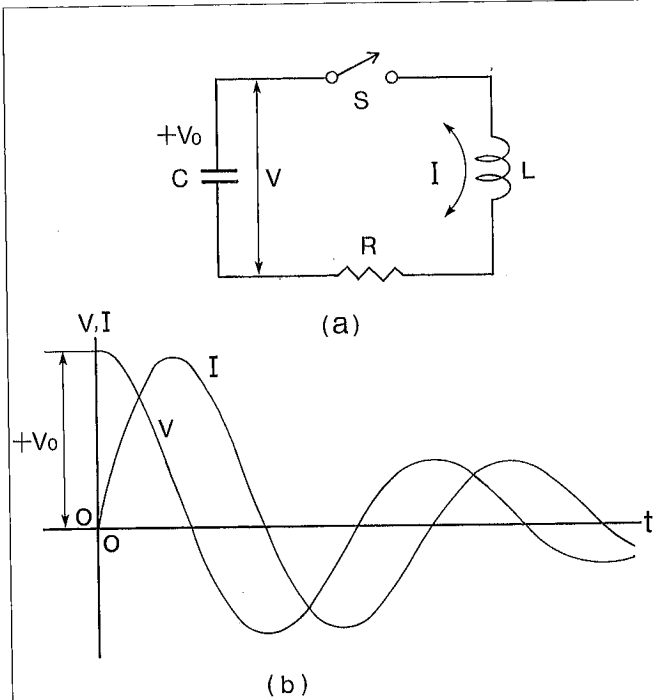
Unlike when the core is fixed outside the coils (i.e., the core has no influence on the coils), the approach of the core results in an increase in combined inductance L , as well as movement of the magnetic flux near the coils. This increase in inductance and the movement of the flux naturally affect the discharge current and recharge voltage.

Generally speaking, it is expected that the total recharge voltage will decrease because this system produces mechanical output as the core moves. However, through a series of experiments, it was discovered that results depend on the magnetic configuration of the coils used in the circuit. In other words, for a certain kind of magnetic field, the opposite result could occur—an increase in the average current and recharge voltage. To confirm the above observations, an experiment was conducted, which is described in the following section.

III. INCREASED VOLTAGE PHENOMENON IN A *LCR* CIRCUIT

A. Experimental method

The experimental setup is schematically shown in Fig. 4. Ferromagnetic cores M_1 and M_2 are attached to the rotor,



which is driven by a dc motor. The cores can be rotated at various speeds, with the speed of the axis being measured by a tachometer. Four electromagnets, L₁, L₂, L₃, and L₄ are connected in series and placed two-by-two in the stators facing each other. The number of turns, inductance (at 1 kHz), and dc inductance of the coils are, respectively, 169, 7.76 mH, and 1.22 Ω. The magnetic field of the electromagnets facing each other, L₁-L₃ and L₂-L₄, can either be attracting (i.e., N-S, N-S) or opposing (i.e., N-S, S-N). The former state will be called the "attracting mode" and the latter the "opposing mode." Placed between the two stators, each containing two electromagnets, is a rotor with two ferromagnetic cores. The specific positions of the electromagnets and the cores are schematically shown in Fig. 5.

At a certain distance between the coil and the core, combined inductance is maximized. This position of the core will be referred to as the "reference point." The reference point

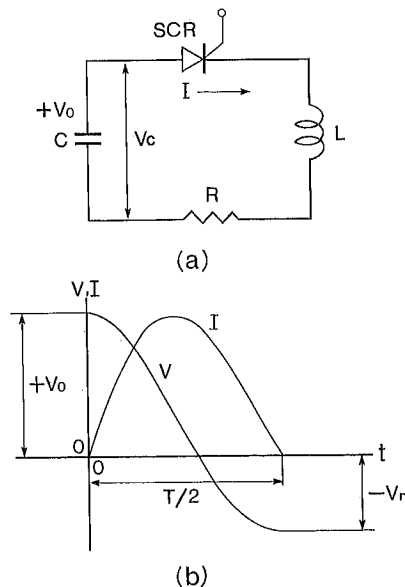


FIG. 2. (a) LCR Circuit with a SCR instead of a switch. (b) Half-cycle voltage and current oscillations of the circuit.

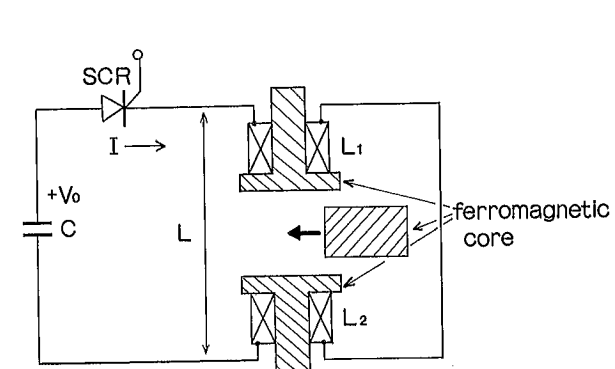


FIG. 3. LCR circuit with two coils and a movable ferromagnetic core inserted between the coils.

will vary slightly, depending on the direction of the magnetic fields. The reference point is exactly aligned with the electromagnets when the magnetic fields are attracting, and slightly displaced when they are opposing.

Figure 6 shows how the inductance of the electromagnet, measured by an LCR meter, is related to the displacement of

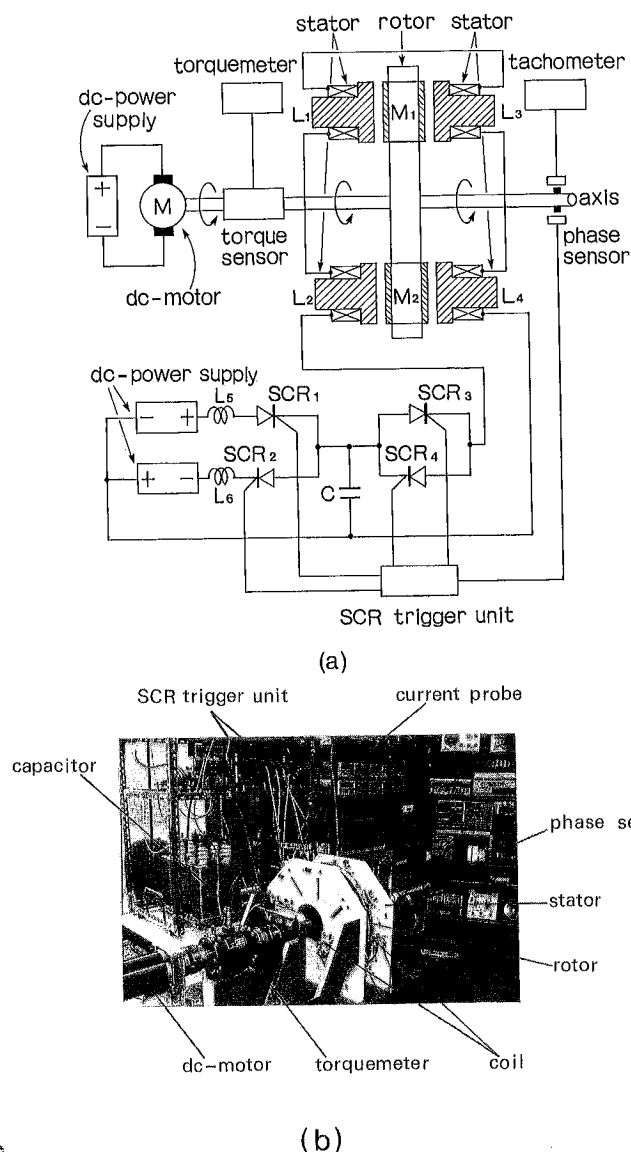


FIG. 4. Experimental setup.

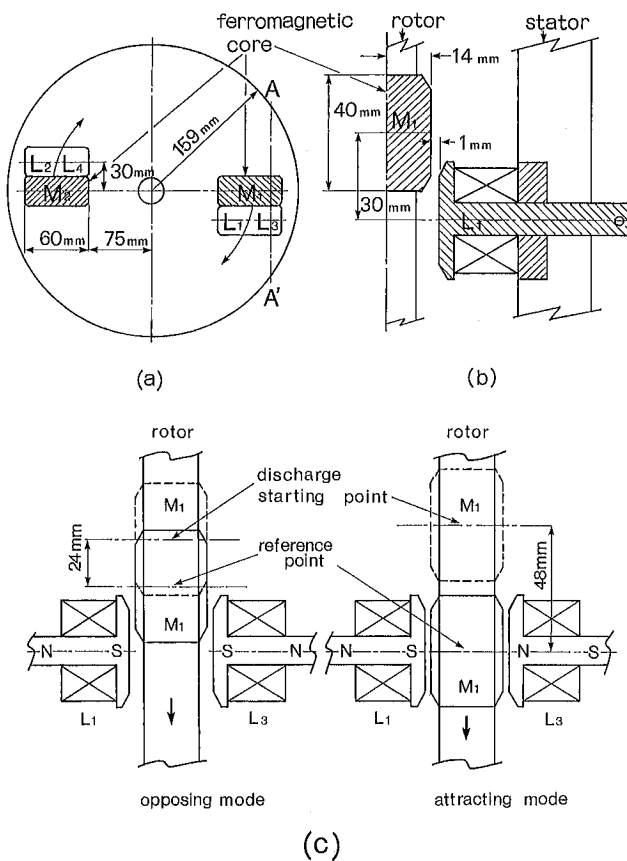


FIG. 5. Specific geometry of the electromagnets and the ferromagnetic core: (a) as viewed from the direction of the axis, (b) the cross section when cut from line A to A', and (c) the stator and rotor at the reference points and the discharge-initiation points.

the core from the electromagnet. The force of attraction (torque) between the electromagnet and the core is also indicated. This *L-d* (inductance-displacement) curve shows that the inductance gradually increases as the core approaches the magnet, reaching a maximum at the reference point (*d*=0). It can be seen from the figure that the rate of change of inductance depends on the magnetic fields and is greater in the attracting mode.

A discharge is initiated at the distance from the reference point, at which the core can experience the maximum force

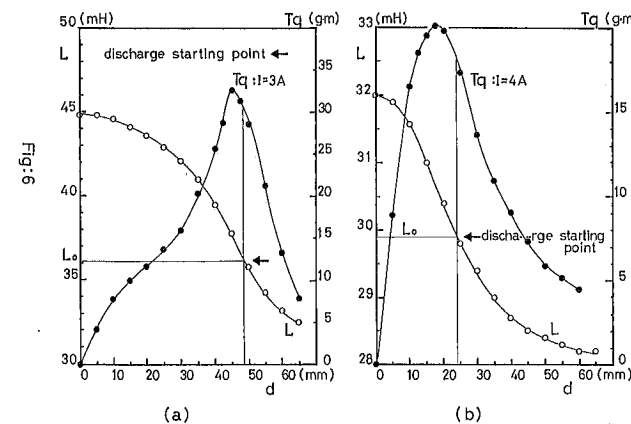


FIG. 6. Combined inductance (*L*) and torque (*Tq*) between the electromagnet and the ferromagnetic core. The torque was measured under conditions of constant current. Values of *L* and *Tq* for (a) the attracting mode, and (b) the opposing mode.

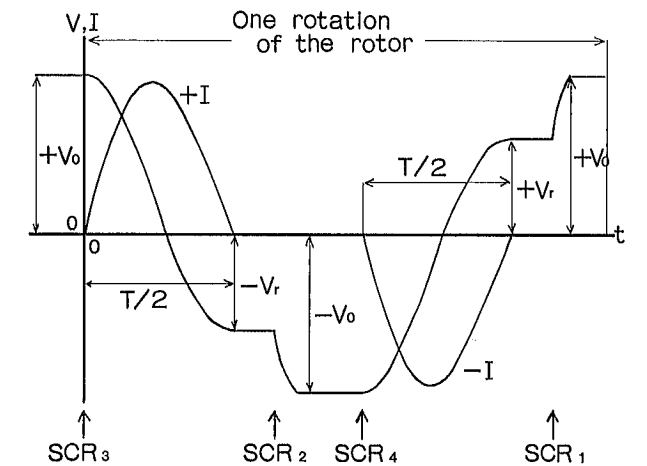


FIG. 7. Voltage and current changes during one rotation of the rotor. The SCR turn-on points are also indicated.

of attraction, i.e., *L*=36.1 mH in the attracting mode and *L*=29.9 mH in the opposing mode. The discharge is completed before the core reaches the reference point, indicating that the rotor does not receive a negative torque from the discharging coil.

The serial operation of the system (Fig. 7) is as follows:

- (1) The capacitor is charged to +V₀.
- (2) When the ferromagnetic cores approach the electromagnets, SCR 3 is turned on and the capacitor is recharged to -V_r.
- (3) SCR 2 is then turned on and the capacitor is charged to -V₀.
- (4) The same cycle is repeated with the opposite current by turning on SCR 4 and SCR 1 in succession (coils L₅ and L₆ are used for protection from an overcharge current).

Thus, for each half rotation, the positive and negative discharges are alternately repeated.

The positive and negative discharges are not completely symmetrical. The conditions for each discharge are not exactly the same due to the particular structure of the experimental device, such as the shape, size, and position of the core. This inevitably causes a slight difference in the recharge-voltage efficiency between two opposite discharges. This condition, however, applies to all cases examined.

Capacitance *C* is set at 15.87 μF, and the initial voltage V₀ at ±240 V. The capacitor voltage was measured by a high-voltage probe of 200 MΩ (dc~15 kHz) impedance, and the current was measured by a clamp-type probe. Wave forms of the capacitor voltage and the discharge current were simultaneously recorded using a digital-storage oscilloscope with a vertical resolution of 8 bits (1/256; the scale ranges from -320 to +320 V). Because of the dispersion of the data, eight measurements of the positive and negative discharges were recorded, and the averages were then examined. The estimated values were calculated by a computer connected directly to the oscilloscope.

B. Estimation factors

One estimation factor is the return-voltage rate, designated as *r*. This rate is defined according to the initial voltage V₀ and the recharge voltage -V_r as

$$r = |V_r|/|V_0| \quad (1)$$

Another factor is the apparent resistance R , derived from V_0 , V_r , and current I . This value can be deduced using the following procedure. First, the energy relationship before ($t=0$) and after ($t=T/2$) the discharge is known as

$$E_0 = E_1 + E_r \quad (2)$$

where E_0 is the initial electrostatic energy of the capacitor, E_1 the recharged electrostatic energy of the capacitor, and E_r the internal energy loss.

The values E_0 , E_1 , and E_r are expressed, respectively, as

$$E_0 = CV_0^2/2, \quad (3)$$

$$E_1 = CV_r^2/2, \quad (4)$$

and

$$E_r = R \int_0^{T/2} I^2 dt. \quad (5)$$

From these equations, R is written as

$$R = C(V_0^2 - V_r^2) / \left(2 \int_0^{T/2} I^2 dt \right) \quad (6)$$

This value of R indicates not only the resistance measured in the dc current but also includes the eddy current loss and hysteresis loss, as well as the effect of back EMF (electromotive force). These resistances are generated when an inductor interacts with a ferromagnetic core. In short, R can be regarded as the total Joule loss of the inductor measured in the ac system.

As discussed in the previous section, the voltage-current wave forms were accurately measured and recorded by a digital storage oscilloscope. From these wave forms, V_0 , V_r , and $(\int I^2 dt)$ can be calculated, and R can thus be estimated.

C. Results

Figures 8 and 9 show the results of the experiments. Return-voltage rate r and apparent resistance R are compared at different rotor speeds from near zero to 400 r/min.

When the magnetic field is in the attracting mode, the value of r follows a monotonic decreasing curve as the rotor speed increases. Correspondingly, resistance R follows an increasing curve. On the other hand, r is found to increase slightly in the opposing mode, with a peak in the 50–100 r/min range, and then gradually decrease at faster speeds. The values of R follow an opposite curve.

IV. MATHEMATICAL ANALYSIS

In order to fully understand the above phenomenon, a computer simulation of the relevant differential equation was made.

To describe the model, inductance L should be replaced by a time-dependent function $L(t)$ that expresses the serial change of L . It is clear that the combined inductance of the

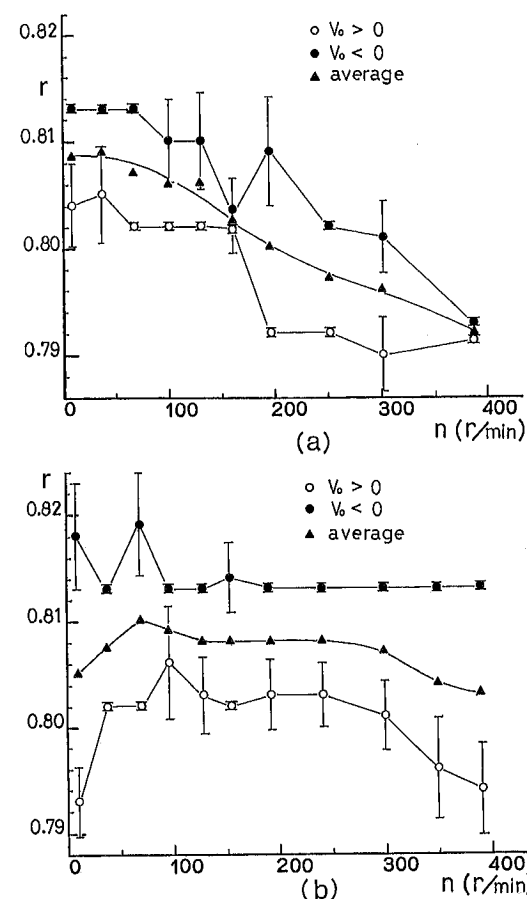


FIG. 8. Results of the experiment—voltage return rate (r) for (a) the attracting mode, and (b) the opposing mode.

electromagnet and the ferromagnetic core is related to the speed at which the ferromagnetic core approaches the electromagnet.

In the experiment, discharge begins when the core experiences the maximum torque, with the core traveling only a very short distance. The change in inductance caused by this motion may be regarded as linear over this narrow range. Thus, the change in inductance can be expressed by the following equation:

$$L = L_0 + \alpha x \quad (\text{H}), \quad (7)$$

where L_0 is the initial inductance of the electromagnet (H), α the rate of inductance change over the distance (H/m), and x the displacement of the core (m).

From $x=vt$, where v is the speed of the core and t is the duration of discharge, Eq. (7) can be transformed as follows:

$$L = L_0 + \alpha vt \quad (\text{H}). \quad (8)$$

The value αv (H s⁻¹) in this equation is the rate of inductance change over time. Since α is constant (unique to each coil), this rate is proportional to v .

From Faraday's law, the voltage across coil V_1 under conditions of changing inductance $L(t)$, is expressed as

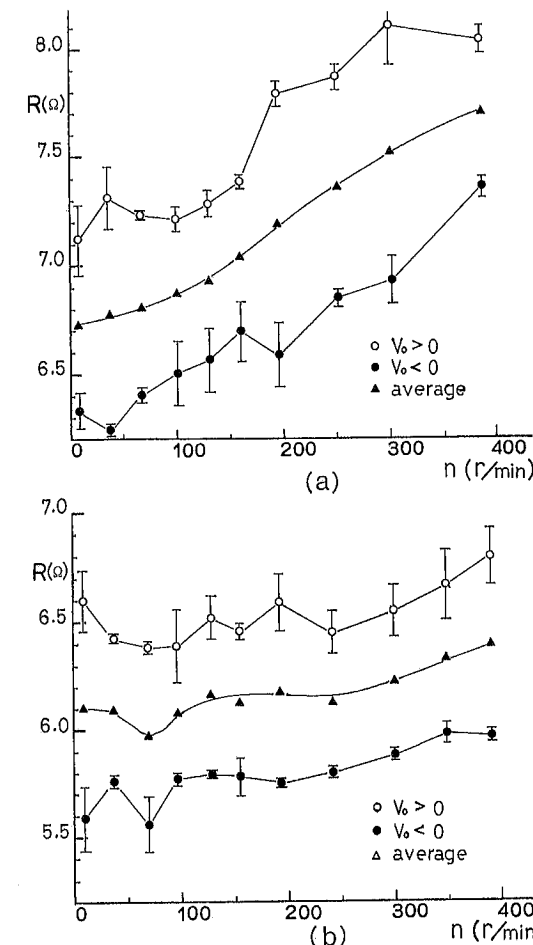


FIG. 9. Results of the experiment—apparent resistance (R) for (a) the attracting mode, and (b) the opposing mode.

$$V_1 = d(LI)/dt = d[(L_0 + \alpha vt)I]/dt = (L_0 + \alpha vt)(dI/dt) + \alpha vI, \quad (9)$$

where v is a constant.

The terminal voltage of capacitor V_c is expressed as

$$V_c = (1/C) \int Idt, \quad (10)$$

where C is a capacitance.

By applying Kirchhoff's law to V_1 , V_c , and the voltage resulting from resistance R and current I , the following equation is obtained:

$$(L_0 + \alpha vt)(dI/dt) + (1/C) \int Idt + (R + \alpha v)I = 0. \quad (11)$$

This differential equation can be solved by using the Runge-Kutta process with initial conditions $t=0$, $V=V_0$, and $I=0$.

Figure 10 and Table I show the computer simulation of the solution of this equation for the specific values of V_0 , C , α , v , and R , as compared with the experimental results. The value of α is obtained from the slope of the L - d curve (Fig. 6) at L_0 . The value of v is calculated from the core's rpm (revolution per minute) and the traveling distance at one rotation.

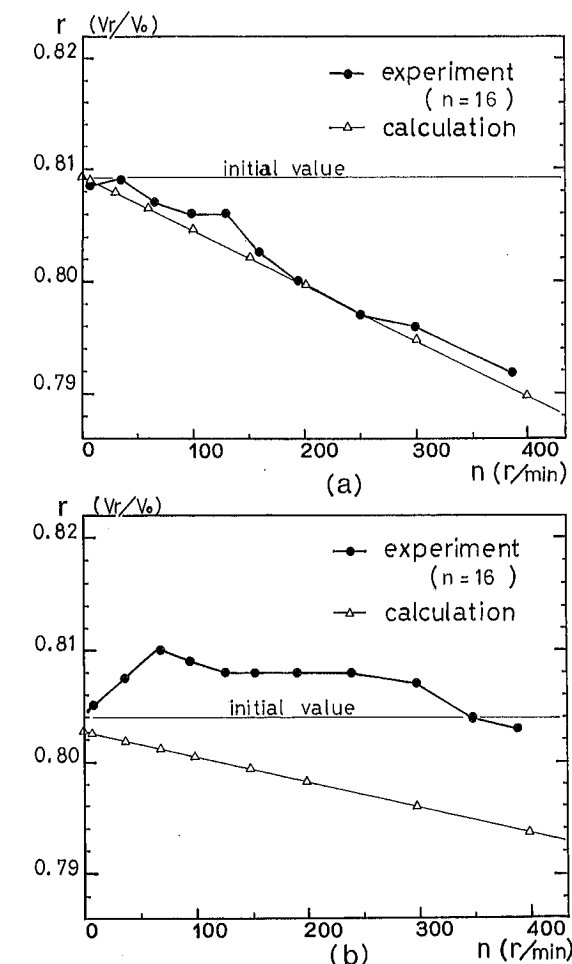


FIG. 10. The computer simulation of the Eq. (11), as compared with the experimental results: (a) the attracting mode, and (b) the opposing mode.

The simulation predicts that the recharge voltage will decrease as the core speed increases. This conforms well with the experimental results in the attracting mode. In the opposing mode, however, certain discrepancies can be found between the simulation and the actual results.

V. DISCUSSION

The mathematical analysis in the previous section reveals that the recharge voltage decreases as the core speed increases. With a constant core speed, it predicts that the recharge voltage will decrease as α increases.

The reason for the decrease in the recharge voltage can be given as follows. The displacement of the core during discharge means there is a mechanical output in the system. It is reasonable to conclude that the mechanical motion of the core is compensated for by a decrease in recharge voltage. The cause of the increase in the apparent resistance R is

	V_0 (V)	C (μ F)	L_0 (mH)	R (Ω)	α (H/m)	v (m/sec)
Fig.10(a) attracting mode	240	15.87	36.1	6.41	0.285	$0.01665 \times n$
Fig.10(b) opposing mode	240	15.87	29.9	6.06	0.119	$0.01665 \times n$

n (r/min)

considered to be the back EMF generated by the movement of the core. For a constant rotor speed, it is clear that a coil with a large α has a large attracting force.

However, the results differ in the case of the opposing mode. Though α is positive, r increases over the range up to a certain speed. After the peak, r decreases slightly but remains greater than the initial value.

These results can be explained from the assumption that the complex movement of the flux could generate a positive EMF: the increase in the recharge voltage is due to an EMF in the same direction as the discharge current, different from the back EMF caused by Faraday's law.

The past controversy concerning electromagnetic induction might shed some light on this viewpoint. On this topic, several authors have stated that the motional EMF caused by the cutting of the magnetic flux and the induced EMF caused by Faraday's law were independent phenomena.^{4,5} These two different types of EMF are generally expressed by the following equation:

$$V = -d\Phi/dt + \int_0^l (B \times v) dl. \quad (12)$$

It can be postulated that these two types of EMF have contradicting effects within the coil, and that the motional EMF has a positive effect on the recharge voltage over a certain range of core speed.

This hypothesis seems to be consistent with the results, but is also highly speculative. It would be necessary to confirm its validity through further experimentation.

VI. CONCLUSIONS

In this paper, the behavior of an *LCR* circuit with a movable ferromagnetic core was discussed. The increase in the

inductance of the coil, which is caused by the attraction of the core during discharge, yielded the following results.

- (1) The recharge voltage is generally smaller when the core moves than when it is stationary. The decrease in the recharge voltage depends on the rate of change of the inductance. The simulation based on the theoretical equation confirmed the experimental results.
- (2) When applying opposing magnetic fields to the facing coils, an increase in the recharge voltage can be observed in an electrically closed *LCR* circuit. The apparent resistance of the coil decreased correspondingly.
- (3) It can be postulated that the complex movement of magnetic flux generates a positive EMF, but the cause of the voltage increase is not clear.

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¹J. J. Blakley, IEEE Trans. Magn. MAG-19, 1570 (1983).

²B. H. Smith, IEE 114, 1707 (1967).

³S. Kikuchi, Y. Sakamoto, and K. Murakami, IEEE Trans. Magn. MAG-20, 1792 (1984).

⁴G. Cohn, Electrical Eng. 68, 441 (1949).

⁵P. Moon and D. Spencer, J. Franklin Institute 260, 213 (1955).

⁶J. C. West and B. V. Jayawant, Institution Electrical Eng. 109A, 292 (1962).

⁷G. W. Swift, IEEE Trans. Power Apparatus Systems PAS-88, 42 (1989).