2. The Cosmological Argument

The cosmological argument (Platon, Philoponos, Aquinas etc.) states that a First Cause of the universe must exist, since no causal chains are finite and do not contain loops. Insofar it is based on the simple fact that any component of an ordered and circle-free finite graph is a tree and thus must have a first vertex. To apply this to a pre-Einstein universe one must add the assumptions that matter does not by itself emerge out of the vacuum, that no motion can occur out of rest, and that initial or primordial matter is at rest. It follows that a First Mover must have initiated all cosmic motion at the moment the universe comes into existence. Since it is not by itself a natural phenomenon, such a mighty cause can only be a god.

However, this argument does not remain valid if the universe has origined from a Big Bang. If the universe expands out of a zero-volume point at $t = 0$, there no causal relation can connect an event at some time $t > 0$ of the existing universe by an event $t < 0$ before the Big Bang. In the watchmaker analogy, the watch could not have been wound up.

We nevertheless embrace the result of the argument, as far as it is restricted to the types of steady-state universes for which it was originally intended. It goes without saying that anything beyond the three-dimensional Euclidean space was out of imagination for the medieval scholar. Before Einstein, time was considered absolute and independent of space and matter. A physical explanation for a universe emerging out of nothing was unthinkable and incompatible with the mechanics of their time, may it be Aristotelian, Galileian or Newtonian. The initial singularity of an Einstein-Friedman universe is, however, a distinctive topological feature of the manifold itself. We assume therefore, in accordance with the cosmological argument, that a finite Aristotelian universe, which manifold can be desribed by a compact subset of $\mathbb{R}^3$ homeomorphic to a ball (a 3-cell), has one and only one god.

3. The Main Theorem

Let U be a unviverse with underlying manifold M_U. By $\Theta(U)$ we denote the number of gods in the universe. We postulate the following axioms.

From the cosmological argument we obtain

Axiom 1. The number of gods in a 3-cell is one.

Gods are eternal, invariable and do not depend on the evolution of the cosmos under the laws of physics. Changing the latter would have had no influence on the existence of gods. Since the laws of physics are continuous, if M_0 and M_1 are the underlying manifolds of the spatial universe at some time t_0 and t_1 (in comoving coordinates), respectively, they must be homotopy equivalent. We therefore obtain

Axiom 2. The number of gods is a homotopy invariance.

Clearly, each god only belongs to only one universe. In a multiverse consisting of disjoint unions of universes, the number of gods must therefore satisfy additivity.

Axiom 3. Let U and U' be separated universes with compact manifolds. Then the number of gods in the disjoint union is the sum of the numbers of gods of the parts,

$$\Theta(U \sqcup U') = \Theta(U) + \Theta(U'). \tag{3.1}$$

Theorem 1. *The number of gods in a universe equals the Euler characteristics of the underlying manifold,*

$$\Theta(U) = \chi(M_U).$$

Proof. By the second axiom, the number of gods only depend on the underlying manifold. Thus we can write w.l.o.g. $\Theta(U) = \Theta(M_U)$. Since the $n-$cell is null homotope, by axiom 1 and 2 we obtain

$$\Theta(\mathrm{pt}) = 1. \tag{3.2}$$

Let further M and N be any compact sets. We construct a homotopy transforming $M \cup N$ into separated copies A, B, C of the closure of $M \setminus N$, $M \cap N$, and $M \setminus N$, respectively. Equation (3.1) together with axiom 2 implies

$$\begin{aligned}
\Theta(M \cup N) &= \Theta(A) + \Theta(B) + \Theta(C), \\
\Theta(M) &= \Theta(A) + \Theta(B), \\
\Theta(N) &= \Theta(B) + \Theta(C), \\
\Theta(M \cap N) &= \Theta(B).
\end{aligned}$$

We obtain the inclusion-exclusion principle,

$$\Theta(M \cup N) = \Theta(M) + \Theta(N) - \Theta(M \cap N).$$

By a well-known characterization, the latter together with (3.2) implies $\Theta(M) = \chi(M)$ for all compact manifolds M. □

The product theorem for the Euler characteristics implies that for manifolds M and N

$$\Theta(M \otimes N) = \Theta(M) \cdot \chi(N).$$

In particular, if time is itself an interval $T \subset \mathbb{R}$, we find that the number of gods are independent of time,

$$\Theta(M \otimes T) = \Theta(M).$$

This is compatible with the scholastic view introduced by Boetius in his *Consolations* that god is above time. However, the same formula spells trouble for all theologies which are based on a cyclic conception of time, which is widespread in India (Veda) and among native American religions. Since $\chi(S^1) = 0$ there are no gods in such a universe,

$$\Theta(M \otimes S^1) = 0.$$

4. Universes, Heavens and Hells

The number of gods has come out to be an integral number. This rules out any demi-gods or lower devas, as they are known from Greek or Indian mythology. However, the divine cardinal can get negative; with the obvious interpretation of these gods being devils. By the additivity theorem, components of positive and negative Euler characteristics could cancel each other out. We can safely assume, however, (and there is plenty of support from religious texts) that gods and devils can not have stable coexistence in the same part of the universe. Thus each component contains only gods or devils, but not both. The absolute value of the Euler characteristics of the universe therefore equals the number of supreme and most inferior beings in it, dependent on the sign.

A lot of types of universes are godless. These are all spaces which contain the 1-sphere (circle) as a factor, such as the tori and all products of them with an arbitrary manifold. This also applies if the universe is infinite Euclidean, but has additional warped dimensions, as suggested in string theory. Also, a spherical three-dimensional universe would have no gods. The only non-exotic topology with a positive number of gods are Euclidean spaces, which all contain exactly one god, which is well in accordance with the Jewish-Christian and Islamic tradition.

An interesting theoretical question concerns the topological structure of heaven. The 3-dimensional Eucledian space is suitable, but since souls are immaterial, they are not confined to three-dimensionality. It is unlikely, however, that heaven is a bounded manifold, since there should be no limit in heaven. One possible structure of a monotheistic heaven is the real projective plane. Another is the 2-sphere, but it requires a pair of gods. The 2-spehere would have been a preferred choice of Greek philosophers, since its imbedding in the $\mathbb{R}^3$ is in perfect alignment with the Pythagorean-Platonic idea of a perfect body, which was influential in the early Scholastics. A suitable pair of gods entrenched in the Abrahamitic tradition is Jahwe and Asherah, but apparently the couple broke up some time after the late Bronze age [Finkelstein and Silberman 2001].

Constraints by traditional religion are more relaxed insofar there is no dogma imposing an upper boundary for the number of devils. As in the case of heavens, souls are not restricted to exist only in spaces with dimesion at least three. If hells are two-dimensional closed surfaces with topological genus $g > 1$, then the Euler characteristics is $2 - 2g$, which would correspond to a hell with an even number of $2\,(g-1)$ devils. Such hells can best be envisioned as multiple tori. The double torus in the form of the figure 8 has Euler characteristics -2. Each torus attached decrease the Euler characteristics by a further -2. This suggest that the rings of hell are not concentric, as Dante speculated, but that they are lined up such that the soul transgresses through a complete half ring of hell before entering the next level. This is a more hellish scenario than Dante's concentric model and thus more realistic.

5. Evidences

The topology of the universe could in principle be observed if it is finite and small (respectively old) enough to have light travelled through it. In this case one would observe multiple images of the same constellations of a matter, which for each point source of light takes the form of a circle. However, this method is not discriminative for a large or young universe, where it only yields a lower bound for the size of the univese [Bielewicz and Banday 2011]. A test for infinity of the universe in one or several dimensions can be based on statistical analysis of the temperature fluctiations of the background radiation, which is a remainder of the Big Bang. If one or more dimensions of the space are topologica circles, space remains homogenuous, but isotropy is violated.

Unfortunately, topology is just constrained, but not determined by local curvature. Data from the Wilkinson microwave anisotropy probe suggest that the universe is flat with only 0.5% margin of error [Cornish et al. 2004]. A flat universe can have vanishing total energy consistent with an origin from nothing. An infinite Euclidean space fits the data. Some exotic topologies such as the Poincaré homology sphere and the Picard horn have been claimed consistent with the findings, but for the former this has been challenged [Key et al. 2007]. A recent statistical analysis on the number of infinite dimensions compared the Euclidean space $\mathbb{R}^3$, the 3-torus

T^3 and the manifolds $T^2 \otimes \mathbb{R}$ and $S^1 \otimes \mathbb{R}^2$ [Aslanyan and Manohar 2011]. Only the Euclidean space has a non-vanishing Euler characteristics. The most probable topology of the universe was found to be $T^2 \otimes \mathbb{R}$, which would support the atheist view brought forward by many leading cosmologists.